10 | NUCLEAR PHYSICS

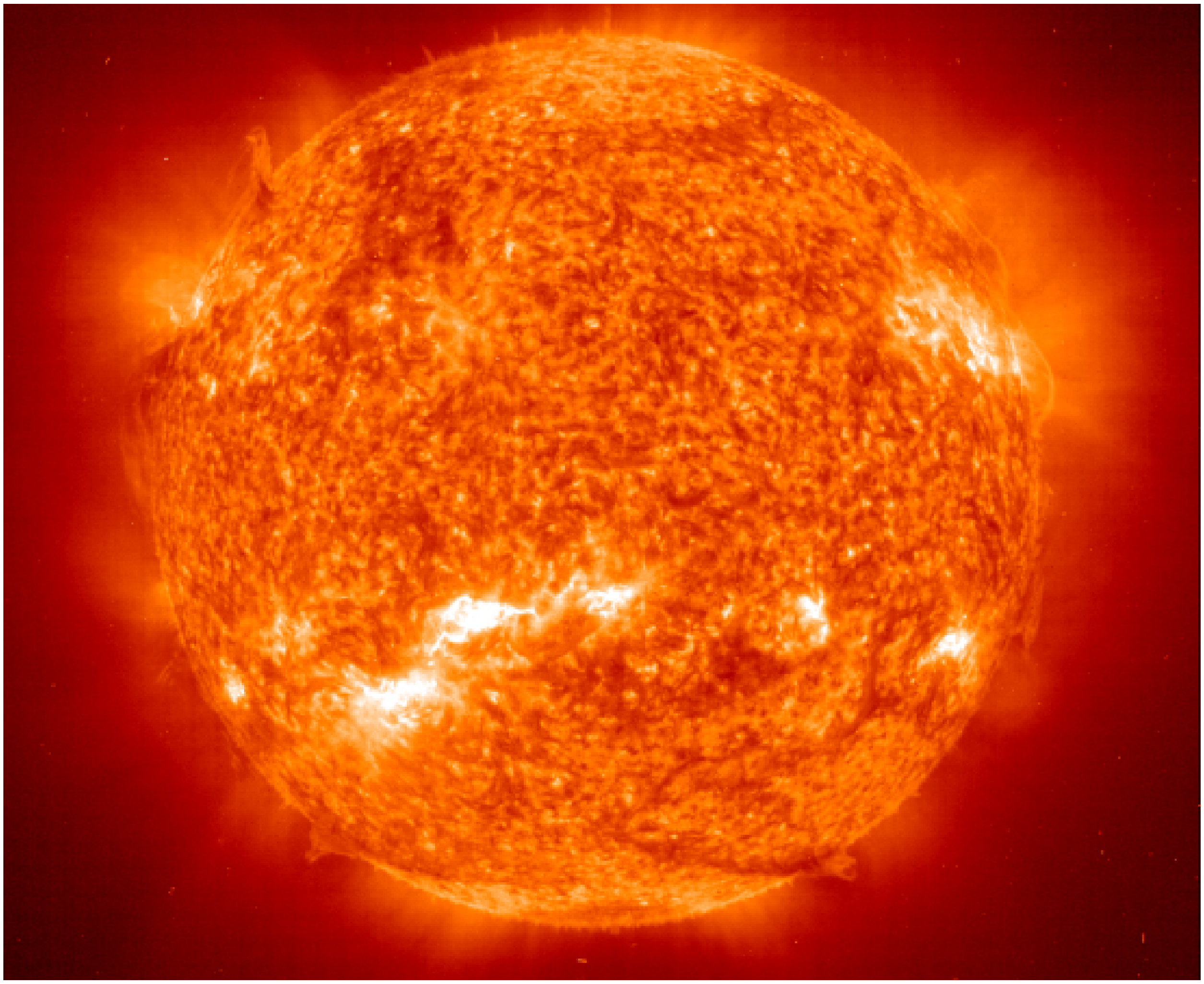

Figure 10.1 The Sun is powered by nuclear fusion in its core. The core converts approximately 10^{38} protons/second into helium at a temperature of 14 million K. This process releases energy in the form of photons, neutrinos, and other particles. (credit: modification of work by EIT SOHO Consortium, ESA, NASA)

Chapter Outline

Introduction

In this chapter, we study the composition and properties of the atomic nucleus. The nucleus lies at the center of an atom, and consists of protons and neutrons. A deep understanding of the nucleus leads to numerous valuable technologies, including devices to date ancient rocks, map the galactic arms of the Milky Way, and generate electrical power.

The Sun is the main source of energy in the solar system. The Sun is 109 Earth diameters across, and accounts for more than 99% of the total mass of the solar system. The Sun shines by fusing hydrogen nuclei—protons—deep inside its interior. Once this fuel is spent, the Sun will burn helium and, later, other nuclei. Nuclear fusion in the Sun is discussed toward the end of this chapter. In the meantime, we will investigate nuclear properties that govern all nuclear processes, including fusion.

10.1 | Properties of Nuclei

Learning Objectives

By the end of this section, you will be able to:

- Describe the composition and size of an atomic nucleus
- Use a nuclear symbol to express the composition of an atomic nucleus
- Explain why the number of neutrons is greater than protons in heavy nuclei
- Calculate the atomic mass of an element given its isotopes

The **atomic nucleus** is composed of **protons** and **neutrons** (**Figure 10.2**). Protons and neutrons have approximately the same mass, but protons carry one unit of positive charge $(+e)$, and neutrons carry no charge. These particles are packed together into an extremely small space at the center of an atom. According to scattering experiments, the nucleus is spherical or ellipsoidal in shape, and about 1/100,000th the size of a hydrogen atom. If an atom were the size of a major league baseball stadium, the nucleus would be roughly the size of the baseball. Protons and neutrons within the nucleus are called **nucleons**.

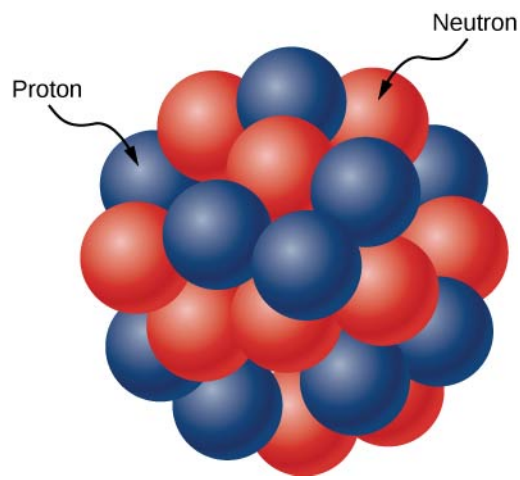

Figure 10.2 The atomic nucleus is composed of protons and neutrons. Protons are shown in blue, and neutrons are shown in red.

Counts of Nucleons

The number of protons in the nucleus is given by the **atomic number**, Z. The number of neutrons in the nucleus is the **neutron number**, N. The total number of nucleons is the **mass number**, A. These numbers are related by

$$A = Z + N. \tag{10.1}$$

A nucleus is represented symbolically by

$$^A_Z\mathrm{X}, \tag{10.2}$$

where X represents the chemical element, *A* is the mass number, and *Z* is the atomic number. For example, $^{12}_{6}\mathrm{C}$ represents the carbon nucleus with six protons and six neutrons (or 12 nucleons).

A graph of the number *N* of neutrons versus the number *Z* of protons for a range of stable nuclei (**nuclides**) is shown in **Figure 10.3**. For a given value of *Z*, multiple values of *N* (blue points) are possible. For small values of *Z*, the number of neutrons equals the number of protons $(N = P),$ and the data fall on the red line. For large values of Z, the number of neutrons is greater than the number of protons $(N > P),$ and the data points fall above the red line. The number of neutrons is generally greater than the number of protons for $Z > 15.$

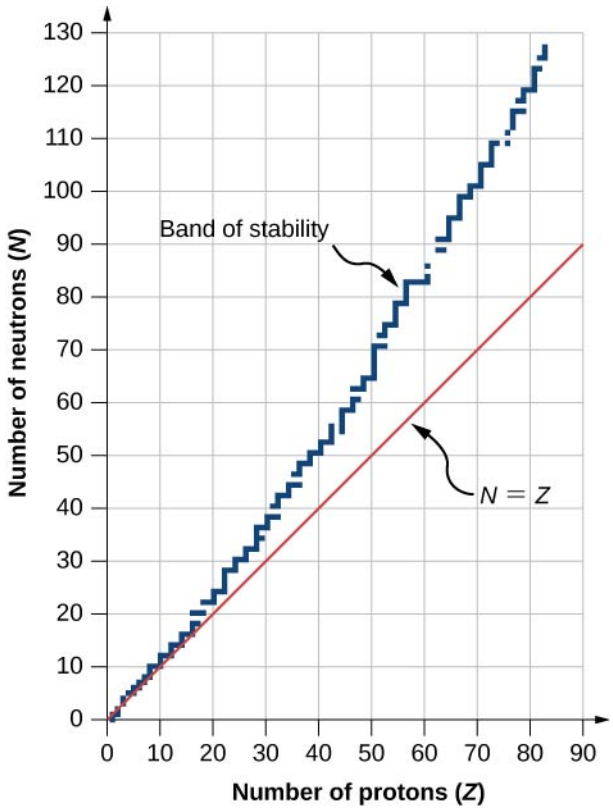

Figure 10.3 This graph plots the number of neutrons *N* against the number of protons *Z* for stable atomic nuclei. Larger nuclei, have more neutrons than protons.

A chart based on this graph that provides more detailed information about each nucleus is given in **Figure 10.4**. This chart is called a **chart of the nuclides**. Each cell or tile represents a separate nucleus. The nuclei are arranged in order of ascending *Z* (along the horizontal direction) and ascending *N* (along the vertical direction).

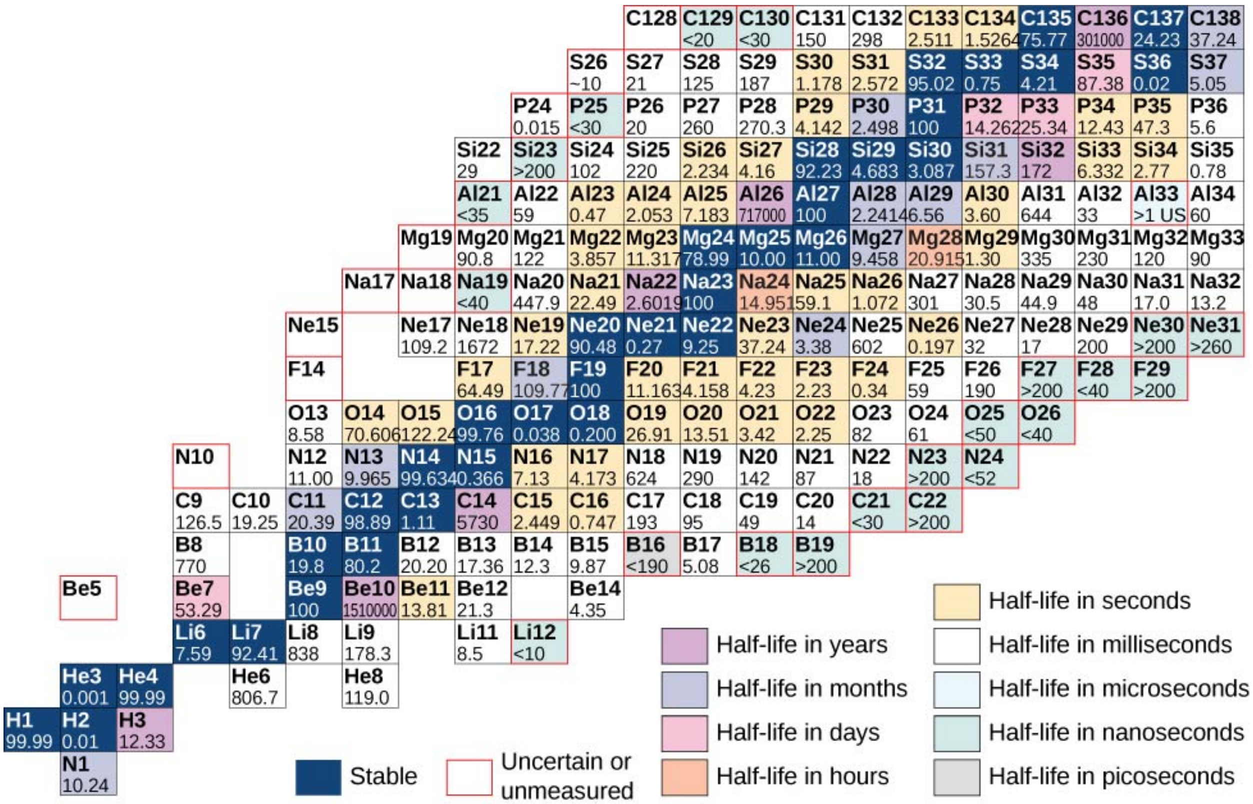

Figure 10.4 Partial chart of the nuclides. For stable nuclei (dark blue backgrounds), cell values represent the percentage of nuclei found on Earth with the same atomic number (percent abundance). For the unstable nuclei, the number represents the half-life.

Atoms that contain nuclei with the same number of protons (Z) and different numbers of neutrons (N) are called **isotopes**. For example, hydrogen has three isotopes: normal hydrogen (1 proton, no neutrons), deuterium (one proton and one neutron), and tritium (one proton and two neutrons). Isotopes of a given atom share the same chemical properties, since these properties are determined by interactions between the outer electrons of the atom, and not the nucleons. For example, water that contains deuterium rather than hydrogen ("heavy water") looks and tastes like normal water. The following table shows a list of common isotopes.

Element	Symbol	Mass Number	Mass (Atomic Mass Units)	Percent Abundance*	Half-life**
Hydrogen	H	1	1.0078	99.99	stable
	^{2}H or D	2	2.0141	0.01	stable
	^{3}H	3	3.0160	–	12.32 y
Carbon	^{12}C	12	12.0000	98.91	stable
	^{13}C	13	13.0034	1.1	stable
	^{14}C	14	14.0032	–	5730 y
Nitrogen	^{14}N	14	14.0031	99.6	stable

Table 10.1 Common Isotopes *No entry if less than 0.001 (trace amount).
**Stable if half-life > 10 seconds.

Element	Symbol	Mass Number	Mass (Atomic Mass Units)	Percent Abundance*	Half-life**
	^{15}N	15	15.0001	0.4	stable
	^{16}N	16	16.0061	–	7.13 s
Oxygen	^{16}O	16	15.9949	99.76	stable
	^{17}O	17	16.9991	0.04	stable
	^{18}O	18	17.9992	0.20	stable
	^{19}O	19	19.0035	–	26.46 s

Table 10.1 Common Isotopes *No entry if less than 0.001 (trace amount).
**Stable if half-life > 10 seconds.

Why do neutrons outnumber protons in heavier nuclei (**Figure 10.5**)? The answer to this question requires an understanding of forces inside the nucleus. Two types of forces exist: (1) the long-range electrostatic (Coulomb) force that makes the positively charged protons repel one another; and (2) the short-range **strong nuclear force** that makes all nucleons in the nucleus attract one another. You may also have heard of a "weak" nuclear force. This force is responsible for some nuclear decays, but as the name implies, it does not play a role in stabilizing the nucleus against the strong Coulomb repulsion it experiences. We discuss strong nuclear force in more detail in the next chapter when we cover particle physics. Nuclear stability occurs when the attractive forces between nucleons compensate for the repulsive, long-range electrostatic forces between all protons in the nucleus. For heavy nuclei $(Z > 15)$, excess neutrons are necessary to keep the electrostatic interactions from breaking the nucleus apart, as shown in **Figure 10.3**.

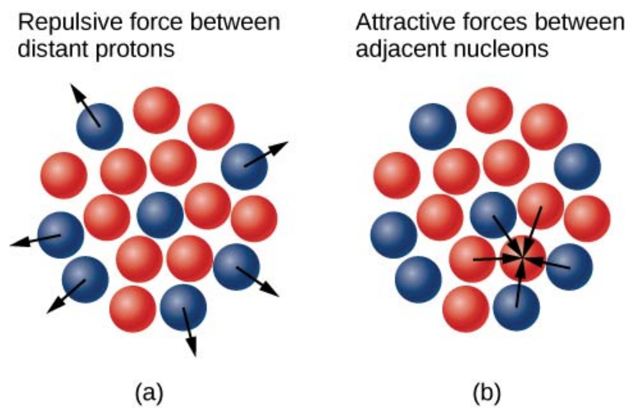

Figure 10.5 (a) The electrostatic force is repulsive and has long range. The arrows represent outward forces on protons (in blue) at the nuclear surface by a proton (also in blue) at the center. (b) The strong nuclear force acts between neighboring nucleons. The arrows represent attractive forces exerted by a neutron (in red) on its nearest neighbors.

Because of the existence of stable isotopes, we must take special care when quoting the mass of an element. For example, Copper (Cu) has two stable isotopes:

$$^{63}_{29}\text{Cu}\,(62.929595\text{ g/mol})\text{ with an abundance of }69.09\%$$

$$^{65}_{29}\text{Cu}\,(64.927786\text{ g/mol})\text{ with an abundance of }30.91\%$$

Given these two "versions" of Cu, what is the mass of this element? The **atomic mass** of an element is defined as the weighted average of the masses of its isotopes. Thus, the atomic mass of Cu is

$m_{Cu} = (62.929595)(0.6909) + (64.927786)(0.3091) = 63.55 \text{ g/mol}$. The mass of an individual nucleus is often expressed in **atomic mass units** (u), where $u = 1.66054 \times 10^{-27} \text{ kg}$. (An atomic mass unit is defined as 1/12th the mass of a ^{12}C nucleus.) In atomic mass units, the mass of a helium nucleus ($A = 4$) is approximately 4 u. A helium nucleus is also called an alpha (α) particle.

Nuclear Size

The simplest model of the nucleus is a densely packed sphere of nucleons. The volume V of the nucleus is therefore proportional to the number of nucleons A, expressed by

$$V = \frac{4}{3}\pi r^3 = kA,$$

where r is the **radius of a nucleus** and k is a constant with units of volume. Solving for r, we have

$$r = r_0 A^{1/3} \tag{10.3}$$

where r_0 is a constant. For hydrogen ($A = 1$), r_0 corresponds to the radius of a single proton. Scattering experiments support this general relationship for a wide range of nuclei, and they imply that neutrons have approximately the same radius as protons. The experimentally measured value for r_0 is approximately 1.2 femtometer (recall that $1 \text{ fm} = 10^{-15} \text{ m}$).

Example 10.1

The Iron Nucleus

Find the radius (r) and approximate density (ρ) of a Fe-56 nucleus. Assume the mass of the Fe-56 nucleus is approximately 56 u.

Strategy

(a) Finding the radius of ^{56}Fe is a straightforward application of $r = r_0 A^{1/3}$, given $A = 56$. (b) To find the approximate density of this nucleus, assume the nucleus is spherical. Calculate its volume using the radius found in part (a), and then find its density from $\rho = m/V$.

Solution

a. The radius of a nucleus is given by

$$r = r_0 A^{1/3}.$$

Substituting the values for r_0 and A yields

$$\begin{aligned} r &= (1.2 \text{ fm})(56)^{1/3} = (1.2 \text{ fm})(3.83) \\ &= 4.6 \text{ fm}. \end{aligned}$$

b. Density is defined to be $\rho = m/V$, which for a sphere of radius r is

$$\rho = \frac{m}{V} = \frac{m}{(4/3)\pi r^3}.$$

Substituting known values gives

$$\rho = \frac{56 \text{ u}}{(1.33)(3.14)(4.6 \text{ fm})^3} = 0.138 \text{ u/fm}^3.$$

Converting to units of kg/m^3, we find

$$\rho = (0.138\ \text{u/fm}^3)(1.66 \times 10^{-27}\ \text{kg/u})\left(\frac{1\ \text{fm}}{10^{-15}\ \text{m}}\right) = 2.3 \times 10^{17}\ \text{kg/m}^3.$$

Significance

a. The radius of the Fe-56 nucleus is found to be approximately 5 fm, so its diameter is about 10 fm, or 10^{-14} m. In previous discussions of Rutherford's scattering experiments, a light nucleus was estimated to be 10^{-15} m in diameter. Therefore, the result shown for a mid-sized nucleus is reasonable.

b. The density found here may seem incredible. However, it is consistent with earlier comments about the nucleus containing nearly all of the mass of the atom in a tiny region of space. One cubic meter of nuclear matter has the same mass as a cube of water 61 km on each side.

10.1 Check Your Understanding Nucleus X is two times larger than nucleus Y. What is the ratio of their atomic masses?

10.2 | Nuclear Binding Energy

Learning Objectives

By the end of this section, you will be able to:

- Calculate the mass defect and binding energy for a wide range of nuclei
- Use a graph of binding energy per nucleon (BEN) versus mass number (A) graph to assess the relative stability of a nucleus
- Compare the binding energy of a nucleon in a nucleus to the ionization energy of an electron in an atom

The forces that bind nucleons together in an atomic nucleus are much greater than those that bind an electron to an atom through electrostatic attraction. This is evident by the relative sizes of the atomic nucleus and the atom $(10^{-15}$ and 10^{-10} m, respectively). The energy required to pry a nucleon from the nucleus is therefore much larger than that required to remove (or ionize) an electron in an atom. In general, all nuclear changes involve large amounts of energy per particle undergoing the reaction. This has numerous practical applications.

Mass Defect

According to nuclear particle experiments, the total mass of a nucleus (m_{nuc}) is *less* than the sum of the masses of its constituent nucleons (protons and neutrons). The mass difference, or **mass defect**, is given by

$$\Delta m = Zm_p + (A - Z)m_n - m_{\text{nuc}} \tag{10.4}$$

where Zm_p is the total mass of the protons, $(A - Z)m_n$ is the total mass of the neutrons, and m_{nuc} is the mass of the nucleus. According to Einstein's special theory of relativity, mass is a measure of the total energy of a system ($E = mc^2$). Thus, the total energy of a nucleus is less than the sum of the energies of its constituent nucleons. The formation of a nucleus from a system of isolated protons and neutrons is therefore an exothermic reaction—meaning that it releases energy. The energy emitted, or radiated, in this process is $(\Delta m)c^2$.

Now imagine this process occurs in reverse. Instead of forming a nucleus, energy is put into the system to break apart the nucleus (Figure 10.6). The amount of energy required is called the total **binding energy (BE)**, E_{b}.

Binding Energy

The binding energy is equal to the amount of energy released in forming the nucleus, and is therefore given by

$$E_b = (\Delta m)c^2. \quad (10.5)$$

Experimental results indicate that the binding energy for a nucleus with mass number $A > 8$ is roughly proportional to the total number of nucleons in the nucleus, A. The binding energy of a magnesium nucleus (^{24}Mg), for example, is approximately two times greater than for the carbon nucleus (^{12}C).

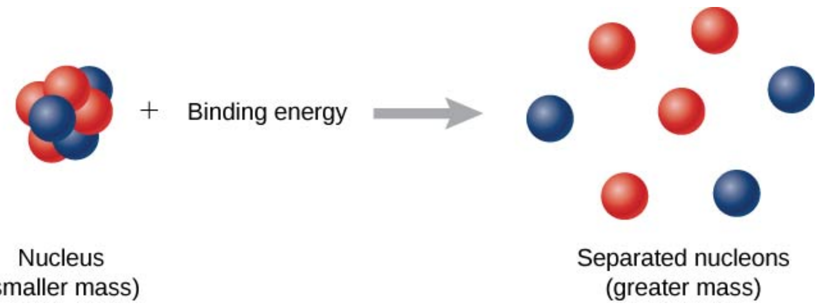

Figure 10.6 The binding energy is the energy required to break a nucleus into its constituent protons and neutrons. A system of separated nucleons has a greater mass than a system of bound nucleons.

Example 10.2

Mass Defect and Binding Energy of the Deuteron

Calculate the mass defect and the binding energy of the deuteron. The mass of the deuteron is $m_D = 3.34359 \times 10^{-27}\,\text{kg}$ or $1875.61\,\text{MeV}/c^2$.

Solution

From **Equation 10.4**, the mass defect for the deuteron is

$$\begin{aligned}\Delta m &= m_p + m_n - m_D \\ &= 938.28\,\text{MeV}/c^2 + 939.57\,\text{MeV}/c^2 - 1875.61\,\text{MeV}/c^2 \\ &= 2.24\,\text{MeV}/c^2.\end{aligned}$$

The binding energy of the deuteron is then

$$E_b = (\Delta m)c^2 = \left(2.24\,\text{MeV}/c^2\right)\left(c^2\right) = 2.24\,\text{MeV}.$$

Over two million electron volts are needed to break apart a deuteron into a proton and a neutron. This very large value indicates the great strength of the nuclear force. By comparison, the greatest amount of energy required to liberate an electron bound to a hydrogen atom by an attractive Coulomb force (an electromagnetic force) is about 10 eV.

Graph of Binding Energy per Nucleon

In nuclear physics, one of the most important experimental quantities is the **binding energy per nucleon (BEN)**, which is defined by

$$BEN = \frac{E_b}{A} \quad (10.6)$$

This quantity is the average energy required to remove an individual nucleon from a nucleus—analogous to the ionization energy of an electron in an atom. If the BEN is relatively large, the nucleus is relatively stable. BEN values are estimated from nuclear scattering experiments.

A graph of binding energy per nucleon versus atomic number *A* is given in **Figure 10.7**. This graph is considered by many physicists to be one of the most important graphs in physics. Two notes are in order. First, typical BEN values range from 6–10 MeV, with an average value of about 8 MeV. In other words, it takes several million electron volts to pry a nucleon from a typical nucleus, as compared to just 13.6 eV to ionize an electron in the ground state of hydrogen. This is why nuclear force is referred to as the "strong" nuclear force.

Second, the graph rises at low *A*, peaks very near iron $(\text{Fe}, \ A = 56)$, and then tapers off at high *A*. The peak value suggests that the iron nucleus is the most stable nucleus in nature (it is also why nuclear fusion in the cores of stars ends with Fe). The reason the graph rises and tapers off has to do with competing forces in the nucleus. At low values of *A*, attractive nuclear forces between nucleons dominate over repulsive electrostatic forces between protons. But at high values of *A*, repulsive electrostatic forces between forces begin to dominate, and these forces tend to break apart the nucleus rather than hold it together.

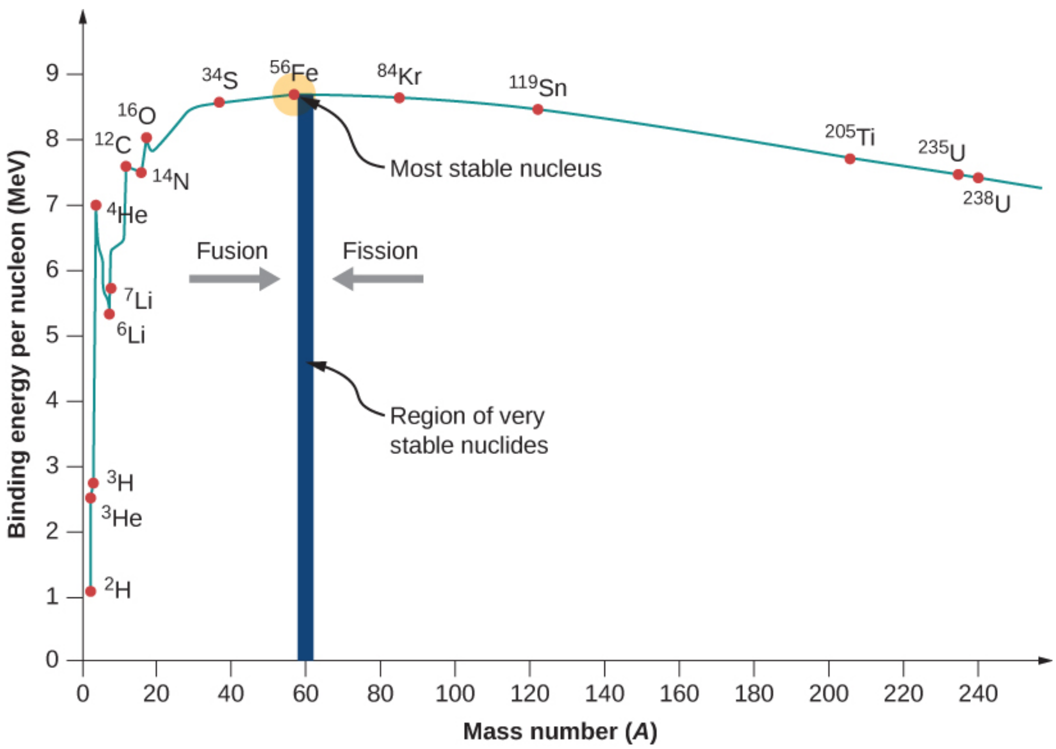

Figure 10.7 In this graph of binding energy per nucleon for stable nuclei, the BEN is greatest for nuclei with a mass near ^{56}Fe. Therefore, fusion of nuclei with mass numbers much less than that of Fe, and fission of nuclei with mass numbers greater than that of Fe, are exothermic processes.

As we will see, the BEN-versus-*A* graph implies that nuclei divided or combined release an enormous amount of energy. This is the basis for a wide range of phenomena, from the production of electricity at a nuclear power plant to sunlight.

Example 10.3

Tightly Bound Alpha Nuclides

Calculate the binding energy per nucleon of an $^4\text{He}\,(\alpha \text{ particle})$.

Strategy

Determine the total binding energy (BE) using the equation $\text{BE} = (\Delta m)c^2$, where Δm is the mass defect. The binding energy per nucleon (BEN) is BE divided by *A*.

Solution

For ^4He, we have $Z = N = 2.$ The total binding energy is

$$\text{BE} = \{[2m_p + 2m_n] - m(^4\text{He})\}c^2.$$

These masses are $m(^4\text{He}) = 4.002602\text{ u}$, $m_p = 1.007825\text{ u},$ and $m_n = 1.008665\text{ u}.$ Thus we have,

$$\text{BE} = (0.030378\text{ u})c^2.$$

Noting that $1\text{ u} = 931.5\text{ MeV}/c^2,$ we find

$$\begin{aligned}\text{BE} &= (0.030378)(931.5\text{ MeV}/c^2)c^2 \\ &= 28.3\text{ MeV}.\end{aligned}$$

Since $A = 4,$ the total binding energy per nucleon is

$$\text{BEN} = 7.07\text{ MeV/nucleon}.$$

Significance

Notice that the binding energy per nucleon for ^4He is much greater than for the hydrogen isotopes $(\text{only} \approx 3\text{ MeV/nucleon}).$ Therefore, helium nuclei cannot break down hydrogen isotopes without energy being put into the system.

10.2 Check Your Understanding If the binding energy per nucleon is large, does this make it harder or easier to strip off a nucleon from a nucleus?

10.3 | Radioactive Decay

Learning Objectives

By the end of this section, you will be able to:

- Describe the decay of a radioactive substance in terms of its decay constant and half-life
- Use the radioactive decay law to estimate the age of a substance
- Explain the natural processes that allow the dating of living tissue using ^{14}C

In 1896, Antoine Becquerel discovered that a uranium-rich rock emits invisible rays that can darken a photographic plate in an enclosed container. Scientists offer three arguments for the nuclear origin of these rays. First, the effects of the radiation do not vary with chemical state; that is, whether the emitting material is in the form of an element or compound. Second, the radiation does not vary with changes in temperature or pressure—both factors that in sufficient degree can affect electrons in an atom. Third, the very large energy of the invisible rays (up to hundreds of eV) is not consistent with atomic electron transitions (only a few eV). Today, this radiation is explained by the conversion of mass into energy deep within the nucleus of an atom. The spontaneous emission of radiation from nuclei is called nuclear **radioactivity** (**Figure 10.8**).

Figure 10.8 The international ionizing radiation symbol is universally recognized as the warning symbol for nuclear radiation.

Radioactive Decay Law

When an individual nucleus transforms into another with the emission of radiation, the nucleus is said to **decay**. Radioactive decay occurs for all nuclei with $Z > 82,$ and also for some unstable isotopes with $Z < 83.$ The decay rate is proportional to the number of original (undecayed) nuclei *N* in a substance. The number of nuclei lost to decay, $-dN$ in time interval *dt*, is written

$$-\frac{dN}{dt} = \lambda N \quad (10.7)$$

where λ is called the **decay constant**. (The minus sign indicates the number of original nuclei decreases over time.) In other words, the more nuclei available to decay, the more that do decay (in time *dt*). This equation can be rewritten as

$$\frac{dN}{N} = -\lambda dt. \quad (10.8)$$

Integrating both sides of the equation, and defining N_0 to be the number of nuclei at $t = 0$, we obtain

$$\int_{N_0}^{N} \frac{dN'}{N} = -\int_0^t \lambda dt'. \quad (10.9)$$

This gives us

$$\ln \frac{N}{N_0} = -\lambda t. \quad (10.10)$$

Taking the left and right sides of the equation as a power of *e*, we have the **radioactive decay law**.

Radioactive Decay Law

The total number *N* of radioactive nuclei remaining after time *t* is

$$N = N_0 e^{-\lambda t} \quad (10.11)$$

where λ is the decay constant for the particular nucleus.

The total number of nuclei drops very rapidly at first, and then more slowly (**Figure 10.9**).

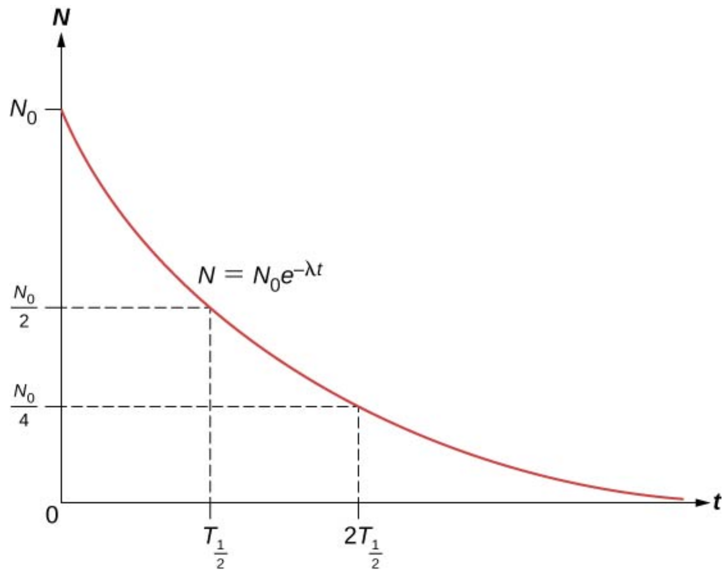

Figure 10.9 A plot of the radioactive decay law demonstrates that the number of nuclei remaining in a decay sample drops dramatically during the first moments of decay.

The **half-life** $(T_{1/2})$ of a radioactive substance is defined as the time for half of the original nuclei to decay (or the time at which half of the original nuclei remain). The half-lives of unstable isotopes are shown in the chart of nuclides in **Figure 10.4**. The number of radioactive nuclei remaining after an integer (n) number of half-lives is therefore

$$N = \frac{N_0}{2^n} \tag{10.12}$$

If the decay constant (λ) is large, the half-life is small, and vice versa. To determine the relationship between these quantities, note that when $t = T_{1/2}$, then $N = N_0/2$. Thus, **Equation 10.10** can be rewritten as

$$\frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}}. \tag{10.13}$$

Dividing both sides by N_0 and taking the natural logarithm yields

$$\ln \frac{1}{2} = \ln e^{-\lambda T_{1/2}} \tag{10.14}$$

which reduces to

$$\lambda = \frac{0.693}{T_{1/2}}. \tag{10.15}$$

Thus, if we know the half-life $T_{1/2}$ of a radioactive substance, we can find its decay constant. The **lifetime** $\bar{T}$ of a radioactive substance is defined as the average amount of time that a nucleus exists before decaying. The lifetime of a substance is just the reciprocal of the decay constant, written as

$$\bar{T} = \frac{1}{\lambda}. \tag{10.16}$$

The **activity** A is defined as the magnitude of the decay rate, or

$$A = -\frac{dN}{dt} = \lambda N = \lambda N_0 e^{-\lambda t}. \quad (10.17)$$

The infinitesimal change dN in the time interval dt is negative because the number of parent (undecayed) particles is decreasing, so the activity (A) is positive. Defining the initial activity as $A_0 = \lambda N_0$, we have

$$A = A_0 e^{-\lambda t}. \quad (10.18)$$

Thus, the activity A of a radioactive substance decreases exponentially with time (**Figure 10.10**).

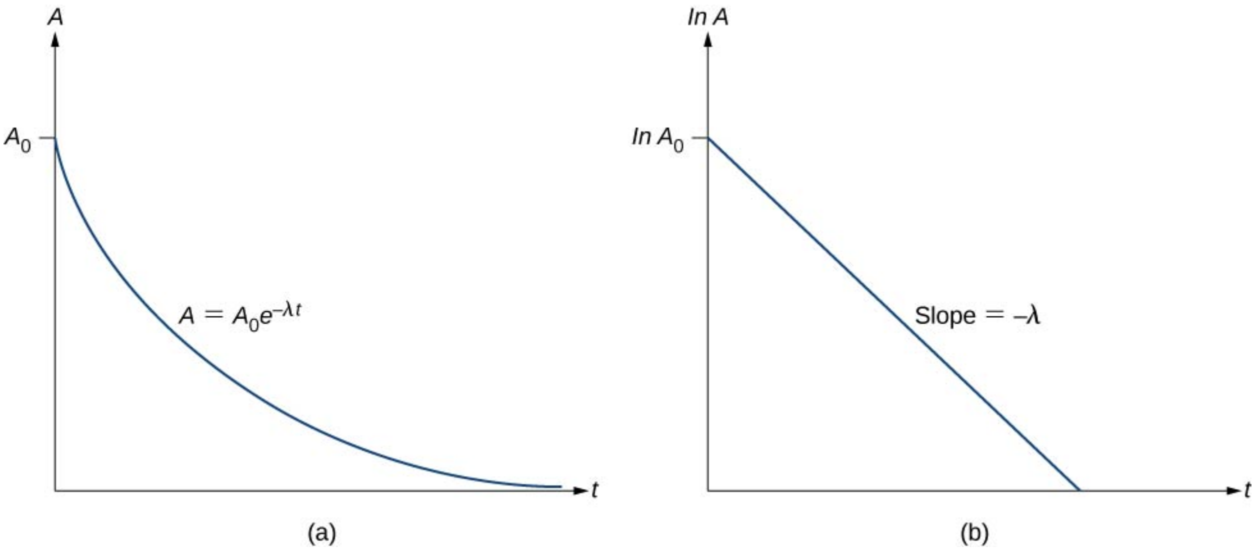

Figure 10.10 (a) A plot of the activity as a function of time (b) If we measure the activity at different times, we can plot ln A versus t, and obtain a straight line.

Example 10.4

Decay Constant and Activity of Strontium-90

The half-life of strontium-90, ${}^{90}_{38}\text{Sr}$, is 28.8 y. Find (a) its decay constant and (b) the initial activity of 1.00 g of the material.

Strategy

We can find the decay constant directly from **Equation 10.15**. To determine the activity, we first need to find the number of nuclei present.

Solution

a. The decay constant is found to be

$$\lambda = \frac{0.693}{T_{1/2}} = \left(\frac{0.693}{T_{1/2}}\right)\left(\frac{1\text{ yr}}{3.16 \times 10^7\text{ s}}\right) = 7.61 \times 10^{-10}\text{ s}^{-1}.$$

b. The atomic mass of ${}^{90}_{38}\text{Sr}$ is 89.91 g. Using Avogadro's number $N_A = 6.022 \times 10^{23}$ atoms/mol, we find the initial number of nuclei in 1.00 g of the material:

$$N_0 = \frac{1.00\text{ g}}{89.91\text{ g}}(N_A) = 6.70 \times 10^{21}\text{ nuclei}.$$

From this, we find that the activity A_0 at $t = 0$ for 1.00 g of strontium-90 is

$$\begin{aligned} A_0 &= \lambda N_0 \\ &= (7.61 \times 10^{-10}\ \text{s}^{-1})(6.70 \times 10^{21}\ \text{nuclei}) \\ &= 5.10 \times 10^{12}\ \text{decays/s}. \end{aligned}$$

Expressing λ in terms of the half-life of the substance, we get

$$A = A_0 e^{-(0.693/T_{1/2})T_{1/2}} = A_0 e^{-0.693} = A_0/2. \quad \textbf{(10.19)}$$

Therefore, the activity is halved after one half-life. We can determine the decay constant λ by measuring the activity as a function of time. Taking the natural logarithm of the left and right sides of **Equation 10.17**, we get

$$\ln A = -\lambda t + \ln A_0. \quad \textbf{(10.20)}$$

This equation follows the linear form $y = mx + b$. If we plot ln A versus t, we expect a straight line with slope $-\lambda$ and y-intercept $\ln A_0$ (**Figure 10.10**(b)). Activity A is expressed in units of **becquerels** (Bq), where one $1\ \text{Bq} = 1\ \text{decay per second}$. This quantity can also be expressed in decays per minute or decays per year. One of the most common units for activity is the **curie (Ci)**, defined to be the activity of 1 g of ^{226}Ra. The relationship between the Bq and Ci is

$$1\ \text{Ci} = 3.70 \times 10^{10}\ \text{Bq}.$$

Example 10.5

What is ^{14}C Activity in Living Tissue?

Approximately 20% of the human body by mass is carbon. Calculate the activity due to ^{14}C in 1.00 kg of carbon found in a living organism. Express the activity in units of Bq and Ci.

Strategy

The activity of ^{14}C is determined using the equation $A_0 = \lambda N_0$, where λ is the decay constant and N_0 is the number of radioactive nuclei. The number of ^{14}C nuclei in a 1.00-kg sample is determined in two steps. First, we determine the number of ^{12}C nuclei using the concept of a mole. Second, we multiply this value by 1.3×10^{-12} (the known abundance of ^{14}C in a carbon sample from a living organism) to determine the number of ^{14}C nuclei in a living organism. The decay constant is determined from the known half-life of ^{14}C (available from **Figure 10.4**).

Solution

One mole of carbon has a mass of 12.0 g, since it is nearly pure ^{12}C. Thus, the number of carbon nuclei in a kilogram is

$$N(^{12}\text{C}) = \frac{6.02 \times 10^{23}\ \text{mol}^{-1}}{12.0\ \text{g/mol}} \times (1000\ \text{g}) = 5.02 \times 10^{25}.$$

The number of ^{14}C nuclei in 1 kg of carbon is therefore

$$N(^{14}\text{C}) = (5.02 \times 10^{25})(1.3 \times 10^{-12}) = 6.52 \times 10^{13}.$$

Now we can find the activity A by using the equation $A = \frac{0.693\,N}{t_{1/2}}$. Entering known values gives us

$$A = \frac{0.693(6.52 \times 10^{13})}{5730\text{ y}} = 7.89 \times 10^{9}\text{ y}^{-1}$$

or 7.89×10^{9} decays per year. To convert this to the unit Bq, we simply convert years to seconds. Thus,

$$A = (7.89 \times 10^{9}\text{ y}^{-1})\frac{1.00\text{ y}}{3.16 \times 10^{7}\text{ s}} = 250\text{ Bq},$$

or 250 decays per second. To express A in curies, we use the definition of a curie,

$$A = \frac{250\text{ Bq}}{3.7 \times 10^{10}\text{ Bq/Ci}} = 6.76 \times 10^{-9}\text{ Ci}.$$

Thus,

$$A = 6.76\text{ nCi}.$$

Significance

Approximately 20% of the human body by weight is carbon. Hundreds of ^{14}C decays take place in the human body every second. Carbon-14 and other naturally occurring radioactive substances in the body compose a person's background exposure to nuclear radiation. As we will see later in this chapter, this activity level is well below the maximum recommended dosages.

Radioactive Dating

Radioactive dating is a technique that uses naturally occurring radioactivity to determine the age of a material, such as a rock or an ancient artifact. The basic approach is to estimate the original number of nuclei in a material and the present number of nuclei in the material (after decay), and then use the known value of the decay constant λ and **Equation 10.10** to calculate the total time of the decay, t.

An important method of radioactive dating is **carbon-14 dating**. Carbon-14 nuclei are produced when high-energy solar radiation strikes ^{14}N nuclei in the upper atmosphere and subsequently decay with a half-life of 5730 years. Radioactive carbon has the same chemistry as stable carbon, so it combines with the ecosphere and eventually becomes part of every living organism. Carbon-14 has an abundance of 1.3 parts per trillion of normal carbon. Therefore, if you know the number of carbon nuclei in an object, you multiply that number by 1.3×10^{-12} to find the number of ^{14}C nuclei in that object. When an organism dies, carbon exchange with the environment ceases, and ^{14}C is not replenished as it decays.

By comparing the abundance of ^{14}C in an artifact, such as mummy wrappings, with the normal abundance in living tissue, it is possible to determine the mummy's age (or the time since the person's death). Carbon-14 dating can be used for biological tissues as old as 50,000 years, but is generally most accurate for younger samples, since the abundance of ^{14}C nuclei in them is greater. Very old biological materials contain no ^{14}C at all. The validity of carbon dating can be checked by other means, such as by historical knowledge or by tree-ring counting.

Example 10.6

An Ancient Burial Cave

In an ancient burial cave, your team of archaeologists discovers ancient wood furniture. Only 80% of the original ^{14}C remains in the wood. How old is the furniture?

Strategy

The problem statement implies that $N/N_0 = 0.80$. Therefore, the equation $N = N_0 e^{-\lambda t}$ can be used to find the product, λt. We know the half-life of ^{14}C is 5730 y, so we also know the decay constant, and therefore the total decay time *t*.

Solution

Solving the equation $N = N_0 e^{-\lambda t}$ for N/N_0 gives us

$$\frac{N}{N_0} = e^{-\lambda t}.$$

Thus,

$$0.80 = e^{-\lambda t}.$$

Taking the natural logarithm of both sides of the equation yields

$$\ln 0.80 = -\lambda t,$$

so that

$$-0.223 = -\lambda t.$$

Rearranging the equation to isolate *t* gives us

$$t = \frac{0.223}{\lambda},$$

where

$$\lambda = \frac{0.693}{t_{1/2}} = \frac{0.693}{5730 \text{ y}}.$$

Combining this information yields

$$t = \frac{0.223}{\left(\frac{0.693}{5730 \text{ y}}\right)} = 1844 \text{ y}.$$

Significance

The furniture is almost 2000 years old—an impressive discovery. The typical uncertainty on carbon-14 dating is about 5%, so the furniture is anywhere between 1750 and 1950 years old. This date range must be confirmed by other evidence, such as historical records.

10.3 Check Your Understanding A radioactive nuclide has a high decay rate. What does this mean for its half-life and activity?

Visit the **Radioactive Dating Game (https://openstaxcollege.org/l/21raddatgame)** to learn about the types of radiometric dating and try your hand at dating some ancient objects.

10.4 | Nuclear Reactions

Learning Objectives

By the end of this section, you will be able to:

- Describe and compare three types of nuclear radiation
- Use nuclear symbols to describe changes that occur during nuclear reactions
- Describe processes involved in the decay series of heavy elements

Early experiments revealed three types of nuclear “rays” or radiation: **alpha (α) rays**, **beta (β) rays**, and **gamma (γ) rays**. These three types of radiation are differentiated by their ability to penetrate matter. Alpha radiation is barely able to pass through a thin sheet of paper. Beta radiation can penetrate aluminum to a depth of about 3 mm, and gamma radiation can penetrate lead to a depth of 2 or more centimeters (Figure 10.11).

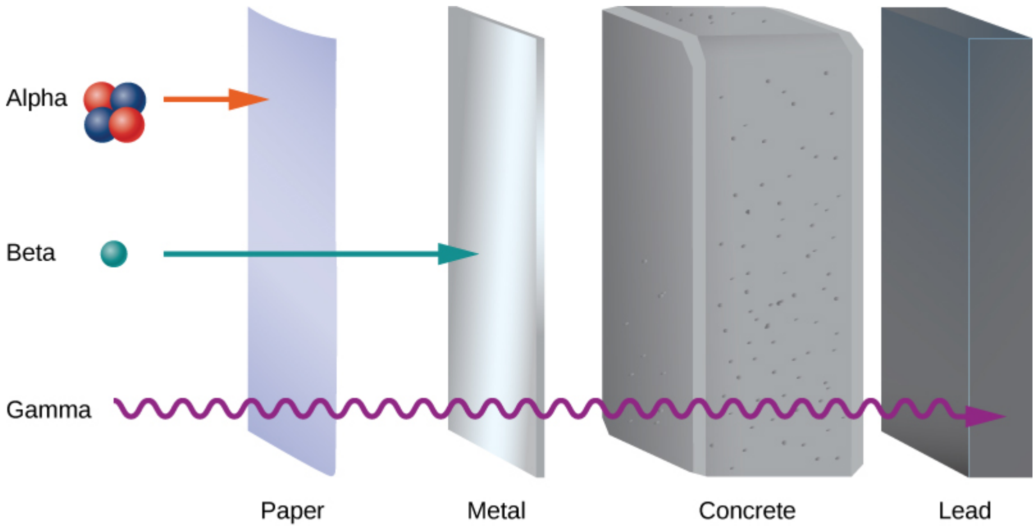

Figure 10.11 A comparison of the penetration depths of alpha (α), beta (β), and gamma (γ) radiation through various materials.

The electrical properties of these three types of radiation are investigated by passing them through a uniform magnetic field, as shown in Figure 10.12. According to the magnetic force equation $\vec{\mathbf{F}} = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$, positively charged particles are deflected upward, negatively charged particles are deflected downward, and particles with no charge pass through the magnetic field undeflected. Eventually, α rays were identified with helium nuclei $\left({}^{4}\text{He}\right)$, β rays with electrons and **positrons** (positively charged electrons or **antielectrons**), and γ rays with high-energy photons. We discuss alpha, beta, and gamma radiation in detail in the remainder of this section.

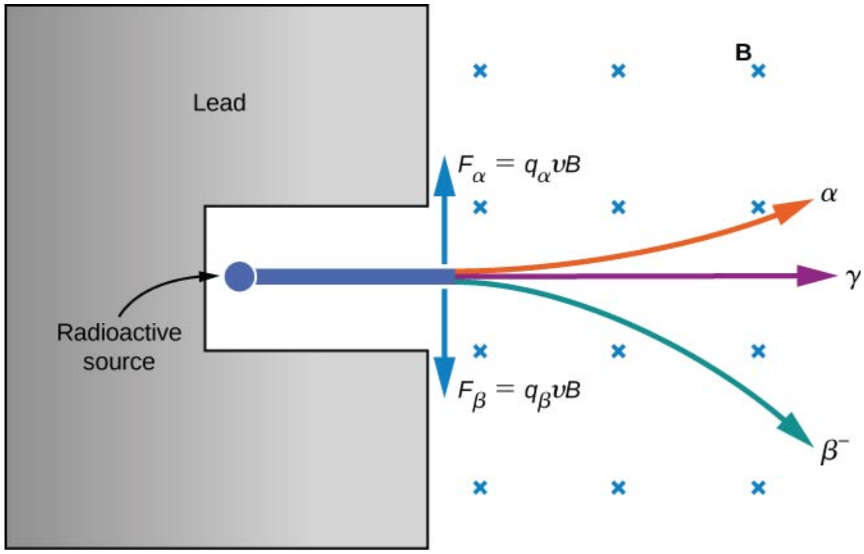

Figure 10.12 The effect of a magnetic field on alpha (α), beta (β), and gamma (γ) radiation. This figure is a schematic only. The relative paths of the particles depend on their masses and initial kinetic energies.

Alpha Decay

Heavy unstable nuclei emit α radiation. In α-particle decay (or **alpha decay**), the nucleus loses two protons and two neutrons, so the atomic number decreases by two, whereas its mass number decreases by four. Before the decay, the nucleus is called the **parent nucleus**. The nucleus or nuclei produced in the decay are referred to as the **daughter nucleus** or daughter nuclei. We represent an α decay symbolically by

$$ {}^{A}_{Z}\text{X} \rightarrow {}^{A-4}_{Z-2}\text{X} + {}^{4}_{2}\text{He} \quad (10.21)$$

where ${}^{A}_{Z}\text{X}$ is the parent nucleus, ${}^{A-4}_{Z-2}\text{X}$ is the daughter nucleus, and ${}^{4}_{2}\text{He}$ is the α particle. In α decay, a nucleus of atomic number Z decays into a nucleus of atomic number $Z-2$ and atomic mass $A-4$. Interestingly, the dream of the ancient alchemists to turn other metals into gold is scientifically feasible through the alpha-decay process. The efforts of the alchemists failed because they relied on chemical interactions rather than nuclear interactions.

Watch alpha particles escape from a polonium nucleus, causing radioactive alpha decay. See how random decay times relate to the half-life. To try a simulation of alpha decay, visit **alpha particles (https://openstaxcollege.org/l/21alphaparvid)**

An example of alpha decay is uranium-238:

$$ {}^{238}_{92}\text{U} \rightarrow {}^{234}_{90}\text{X} + {}^{4}_{2}\text{He}.$$

The atomic number has dropped from 92 to 90. The chemical element with $Z = 90$ is thorium. Hence, Uranium-238 has decayed to Thorium-234 by the emission of an α particle, written

$$ {}^{238}_{92}\text{U} \rightarrow {}^{234}_{90}\text{Th} + {}^{4}_{2}\text{He}.$$

Subsequently, ${}^{234}_{90}\text{Th}$ decays by β emission with a half-life of 24 days. The energy released in this alpha decay takes the form of kinetic energies of the thorium and helium nuclei, although the kinetic energy of thorium is smaller than helium due to its heavier mass and smaller velocity.

Example 10.7

Plutonium Alpha Decay

Find the energy emitted in the α decay of ^{239}Pu.

Strategy

The energy emitted in the α decay of ^{239}Pu can be found using the equation $E = (\Delta m)c^2$. We must first find Δm, the difference in mass between the parent nucleus and the products of the decay.

Solution

The decay equation is

$$^{239}\text{Pu} \rightarrow {}^{235}\text{U} + {}^{4}\text{He}.$$

Thus, the pertinent masses are those of ^{239}Pu, ^{235}U, and the α particle or ^{4}He, all of which are known. The initial mass was $m\left(^{239}\text{Pu}\right) = 239.052157\ \text{u}$. The final mass is the sum

$$\begin{aligned} m\left(^{235}\text{U}\right) + m\left(^{4}\text{He}\right) &= 235.043924\ \text{u} + 4.002602\ \text{u} \\ &= 239.046526\ \text{u}. \end{aligned}$$

Thus,

$$\begin{aligned} \Delta m &= m\left(^{239}\text{Pu}\right) - \left[m\left(^{235}\text{U}\right) + m\left(^{4}\text{He}\right)\right] \\ &= 239.052157\ \text{u} - 239.046526\ \text{u} \\ &= 0.0005631\ \text{u}. \end{aligned}$$

Now we can find E by entering Δm into the equation:

$$E = (\Delta m)c^2 = (0.005631\ \text{u})c^2.$$

We know $1\ \text{u} = 931.5\ \text{MeV}/c^2$, so we have

$$\begin{aligned} E &= (0.005631)\left(931.5\ \text{MeV}/c^2\right)\left(c^2\right) \\ &= 5.25\ \text{MeV}. \end{aligned}$$

Significance

The energy released in this α decay is in the MeV range, many times greater than chemical reaction energies. Most of this energy becomes kinetic energy of the α particle (or ^{4}He nucleus), which moves away at high speed. The energy carried away by the recoil of the ^{235}U nucleus is much smaller due to its relatively large mass. The ^{235}U nucleus can be left in an excited state to later emit photons (γ rays).

Beta Decay

In most β particle decays (or **beta decay**), either an electron (β^-) or positron (β^+) is emitted by a nucleus. A positron has the same mass as the electron, but its charge is $+e$. For this reason, a positron is sometimes called an antielectron. How does β decay occur? A possible explanation is the electron (positron) is confined to the nucleus prior to the decay and somehow escapes. To obtain a rough estimate of the escape energy, consider a simplified model of an electron trapped in a box (or in the terminology of quantum mechanics, a one-dimensional square well) that has the width of a typical nucleus ($10^{-14}\ \text{m}$). According to the Heisenberg uncertainty principle in **Quantum Mechanics**, the uncertainty of the momentum of the electron is:

$$\Delta p > \frac{h}{\Delta x} = \frac{6.6 \times 10^{-34}\ \text{m}^2 \cdot \text{kg/s}}{10^{-14}\,\text{m}} = 6.6 \times 10^{-20}\,\text{kg} \cdot\ \text{m/s}.$$

Taking this momentum value (an underestimate) to be the "true value," the kinetic energy of the electron on escape is approximately

$$\frac{(\Delta p)^2}{2m_e} = \frac{\left(6.6 \times 10^{-20}\ \text{kg} \cdot \text{m/s}\right)^2}{2\left(9.1 \times 10^{-31}\,\text{kg}\right)} = 2.0 \times 10^{-9}\,\text{J} = 12{,}400\,\text{MeV}.$$

Experimentally, the electrons emitted in β^- decay are found to have kinetic energies of the order of only a few MeV. We therefore conclude that the electron is somehow produced in the decay rather than escaping the nucleus. Particle production (annihilation) is described by theories that combine quantum mechanics and relativity, a subject of a more advanced course in physics.

Nuclear beta decay involves the conversion of one nucleon into another. For example, a neutron can decay to a proton by the emission of an electron (β^-) and a nearly massless particle called an **antineutrino** ($\bar{\nu}$):

$${}^{1}_{0}\text{n} \rightarrow {}^{1}_{1}\text{p} + {}^{\ 0}_{-1}\text{e} + \bar{\nu}\ .$$

The notation ${}^{\ 0}_{-1}\text{e}$ is used to designate the electron. Its mass number is 0 because it is not a nucleon, and its atomic number is -1 to signify that it has a charge of $-e$. The proton is represented by ${}^{1}_{1}\text{p}$ because its mass number and atomic number are 1. When this occurs within an atomic nucleus, we have the following equation for beta decay:

$${}^{A}_{Z}\text{X} \rightarrow {}_{Z+1}^{\quad A}\text{X} + {}^{\ 0}_{-1}\text{e} + \bar{\nu}\ . \tag{10.22}$$

As discussed in another chapter, this process occurs due to the weak nuclear force.

Watch **beta decay (https://openstaxcollege.org/l/21betadecayvid)** occur for a collection of nuclei or for an individual nucleus.

As an example, the isotope ${}^{234}_{90}\text{Th}$ is unstable and decays by β^- emission with a half-life of 24 days. Its decay can be represented as

$${}^{234}_{90}\text{Th} \rightarrow {}^{234}_{91}\text{X} + {}^{\ 0}_{-1}\text{e} + \bar{\nu}\ .$$

Since the chemical element with atomic number 91 is protactinium (Pa), we can write the β^- decay of thorium as

$${}^{234}_{90}\text{Th} \rightarrow {}^{234}_{91}\text{Pa} + {}^{\ 0}_{-1}\text{e} + \bar{\nu}\ .$$

The reverse process is also possible: A proton can decay to a neutron by the emission of a positron (β^+) and a nearly massless particle called a **neutrino** (ν). This reaction is written as ${}^{1}_{1}\text{p} \rightarrow {}^{1}_{0}\text{n} + {}^{\ 0}_{+1}\text{e} + \nu$.

The positron ${}^{\ 0}_{+1}\text{e}$ is emitted with the neutrino ν, and the neutron remains in the nucleus. (Like β^- decay, the positron does not precede the decay but is produced in the decay.) For an isolated proton, this process is impossible because the neutron is heavier than the proton. However, this process is possible within the nucleus because the proton can receive energy from other nucleons for the transition. As an example, the isotope of aluminum ${}^{26}_{13}\text{Al}$ decays by β^+ emission with a half-life of $7.40 \times 10^5\,\text{y}$. The decay is written as

$${}^{26}_{13}\text{Al} \rightarrow {}^{26}_{12}\text{X} + {}^{\ 0}_{+1}\text{e} + \nu.$$

The atomic number 12 corresponds to magnesium. Hence,

$$^{26}_{13}\text{Al} \rightarrow ^{26}_{12}\text{Mg} + ^{0}_{+1}\text{e} + \nu.$$

As a nuclear reaction, positron emission can be written as

$$^{A}_{Z}\text{X} \rightarrow _{Z-1}^{\ \ \ A}\text{X} + ^{0}_{+1}\text{e} + \nu. \quad \text{(10.23)}$$

The neutrino was not detected in the early experiments on β decay. However, the laws of energy and momentum seemed to require such a particle. Later, neutrinos were detected through their interactions with nuclei.

Example 10.8

Bismuth Alpha and Beta Decay

The $^{211}_{83}\text{Bi}$ nucleus undergoes both α and β^- decay. For each case, what is the daughter nucleus?

Strategy

We can use the processes described by **Equation 10.21** and **Equation 10.22**, as well as the Periodic Table, to identify the resulting elements.

Solution

The atomic number and the mass number for the α particle are 2 and 4, respectively. Thus, when a bismuth-211 nucleus emits an α particle, the daughter nucleus has an atomic number of 81 and a mass number of 207. The element with an atomic number of 81 is thallium, so the decay is given by

$$^{211}_{83}\text{Bi} \rightarrow ^{207}_{81}\text{Ti} + ^{4}_{2}\text{He}.$$

In β^- decay, the atomic number increases by 1, while the mass number stays the same. The element with an atomic number of 84 is polonium, so the decay is given by

$$^{211}_{83}\text{Bi} \rightarrow ^{211}_{84}\text{Po} + ^{0}_{-1}\text{e} + \bar{\nu}\ .$$

10.4 Check Your Understanding In radioactive beta decay, does the atomic mass number, *A*, increase or decrease?

Gamma Decay

A nucleus in an excited state can decay to a lower-level state by the emission of a "gamma-ray" photon, and this is known as **gamma decay**. This is analogous to de-excitation of an atomic electron. Gamma decay is represented symbolically by

$$^{A}_{Z}\text{X}^* \rightarrow ^{A}_{Z}\text{X} + \gamma \quad \text{(10.24)}$$

where the asterisk (*) on the nucleus indicates an excited state. In γ decay, neither the atomic number nor the mass number changes, so the type of nucleus does not change.

Radioactive Decay Series

Nuclei with $Z > 82$ are unstable and decay naturally. Many of these nuclei have very short lifetimes, so they are not found in nature. Notable exceptions include $^{232}_{90}\text{Th}$ (or Th-232) with a half-life of 1.39×10^{10} years, and $^{238}_{92}\text{U}$ (or U-238) with a half-life of 7.04×10^{8} years. When a heavy nucleus decays to a lighter one, the lighter daughter nucleus can become the parent nucleus for the next decay, and so on. This process can produce a long series of nuclear decays called a **decay series**. The series ends with a stable nucleus.

To illustrate the concept of a decay series, consider the decay of Th-232 series (**Figure 10.13**). The neutron number, *N*, is plotted on the vertical *y*-axis, and the atomic number, *Z*, is plotted on the horizontal *x*-axis, so Th-232 is found at the coordinates $(N, Z) = (142, 90)$. Th-232 decays by α emission with a half-life of 1.39×10^{10} years. Alpha decay decreases the atomic number by 2 and the mass number by 4, so we have

$$^{232}_{90}\text{Th} \rightarrow {}^{228}_{88}\text{Ra} + {}^{4}_{2}\text{He}.$$

The neutron number for Radium-228 is 140, so it is found in the diagram at the coordinates $(N, Z) = (140, 88)$. Radium-228 is also unstable and decays by β^- emission with a half-life of 5.76 years to Actinum-228. The atomic number increases by 1, the mass number remains the same, and the neutron number decreases by 1. Notice that in the graph, α emission appears as a line sloping downward to the left, with both *N* and *Z* decreasing by 2. Beta emission, on the other hand, appears as a line sloping downward to the right with *N* decreasing by 1, and *Z* increasing by 1. After several additional alpha and beta decays, the series ends with the stable nucleus Pb-208.

The relative frequency of different types of radioactive decays (alpha, beta, and gamma) depends on many factors, including the strength of the forces involved and the number of ways a given reaction can occur without violating the conservation of energy and momentum. How often a radioactive decay occurs often depends on a sensitive balance of the strong and electromagnetic forces. These forces are discussed in **Particle Physics and Cosmology**.

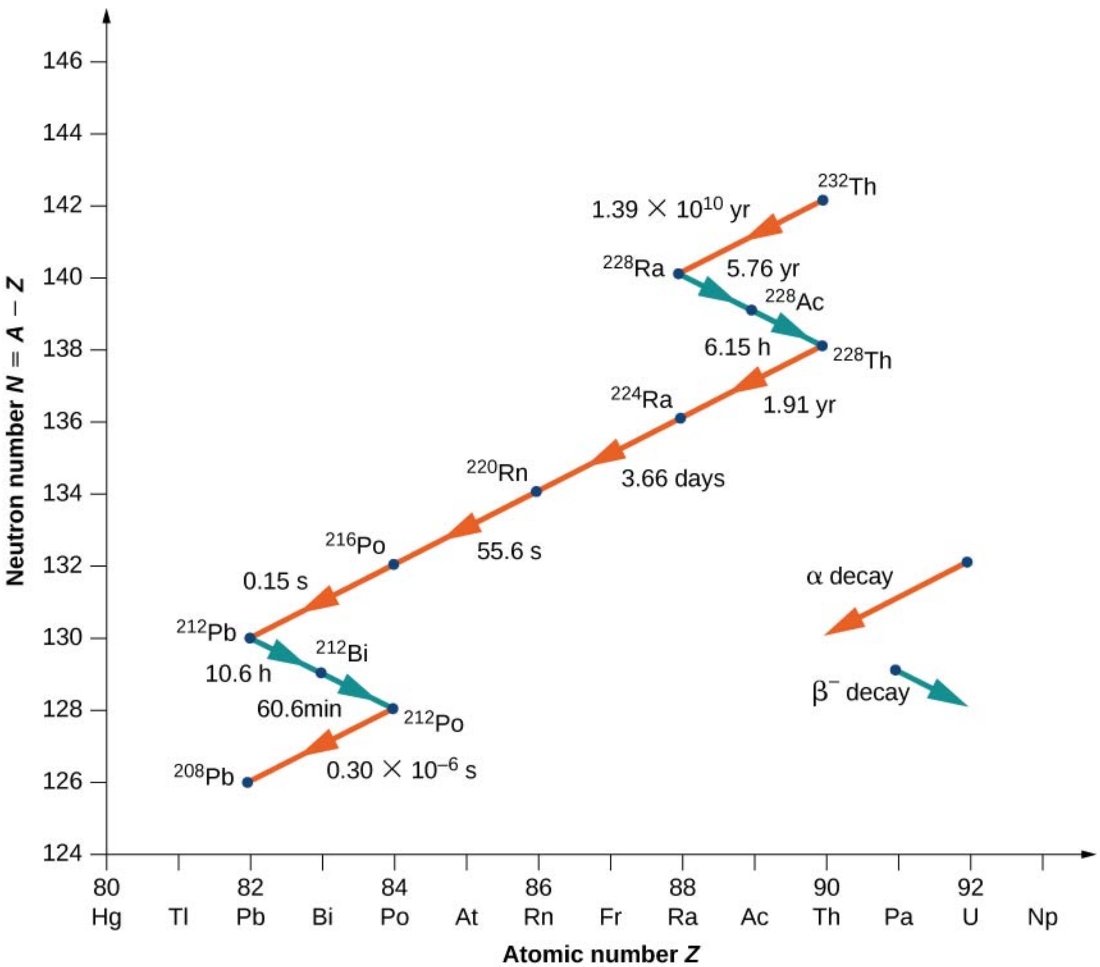

Figure 10.13 In the thorium $^{232}_{90}\text{Th}$ decay series, alpha (α) decays reduce the atomic number, as indicated by the red arrows. Beta (β^-) decays increase the atomic number, as indicated by the blue arrows. The series ends at the stable nucleus Pb-208.

As another example, consider the U-238 decay series shown in **Figure 10.14**. After numerous alpha and beta decays, the series ends with the stable nucleus Pb-206. An example of a decay whose parent nucleus no longer exists naturally is shown in **Figure 10.15**. It starts with Neptunium-237 and ends in the stable nucleus Bismuth-209. Neptunium is called a **transuranic element** because it lies beyond uranium in the periodic table. Uranium has the highest atomic number $(Z = 92)$ of any element found in nature. Elements with $Z > 92$ can be produced only in the laboratory. They most probably also existed in nature at the time of the formation of Earth, but because of their relatively short lifetimes, they have completely decayed. There is nothing fundamentally different between naturally occurring and artificial elements.

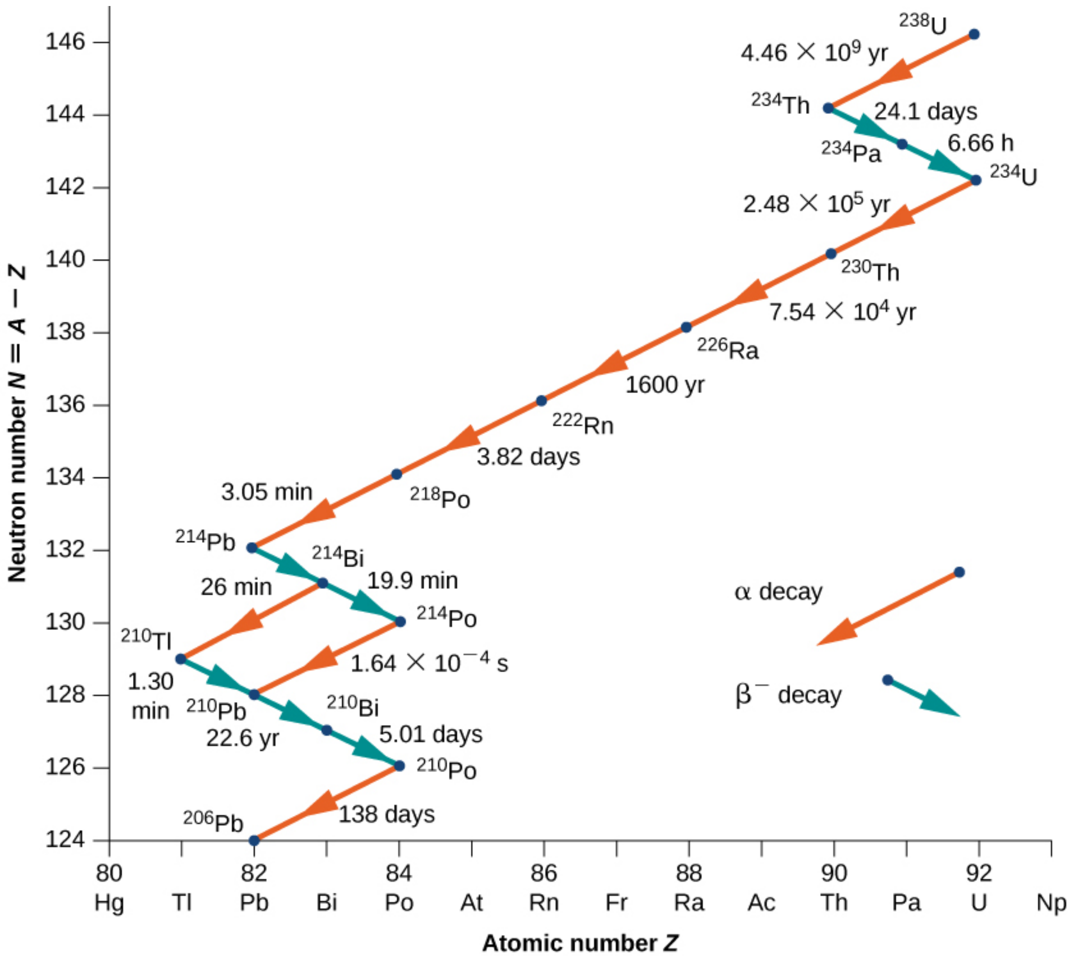

Figure 10.14 In the Uranium-238 decay series, alpha (α) decays reduce the atomic number, as indicated by the red arrows. Beta (β^-) decays increase the atomic number, as indicated by the blue arrows. The series ends at the stable nucleus Pb-206.

Notice that for Bi (21), the decay may proceed through either alpha or beta decay.

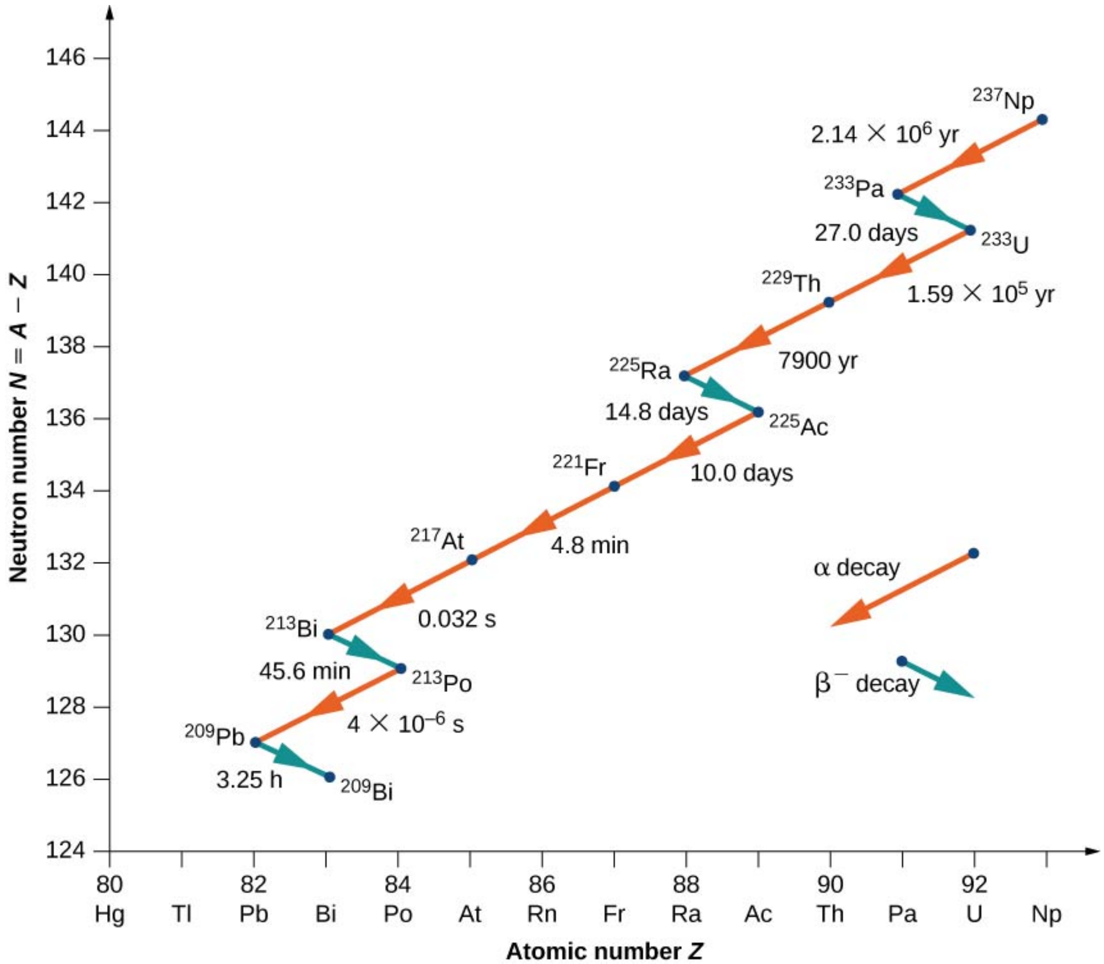

Figure 10.15 In the Neptunium-237 decay series, alpha (α) decays reduce the atomic number, as indicated by the red arrows. Beta (β^-) decays increase the atomic number, as indicated by the blue arrows. The series ends at the stable nucleus Bi-209.

Radioactivity in the Earth

According to geologists, if there were no heat source, Earth should have cooled to its present temperature in no more than 1 billion years. Yet, Earth is more than 4 billion years old. Why is Earth cooling so slowly? The answer is nuclear radioactivity, that is, high-energy particles produced in radioactive decays heat Earth from the inside (**Figure 10.16**).

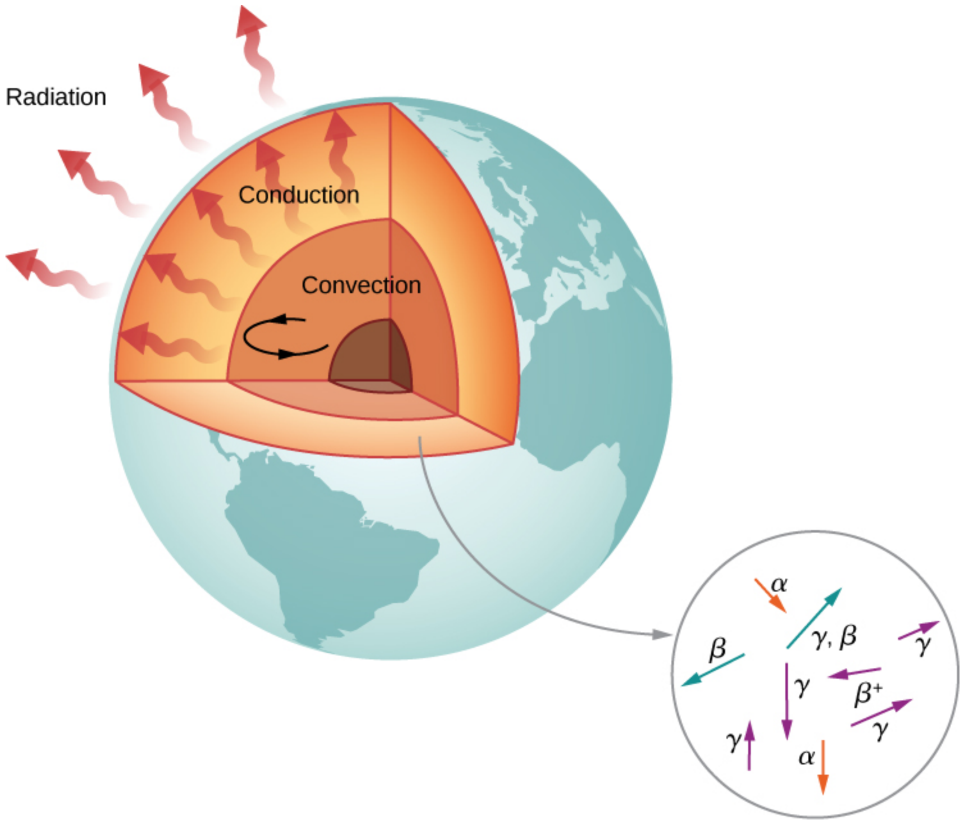

Figure 10.16 Earth is heated by nuclear reactions (alpha, beta, and gamma decays). Without these reactions, Earth's surface would be much cooler than it is now.

Candidate nuclei for this heating model are ^{238}U and ^{40}K, which possess half-lives similar to or longer than the age of Earth. The energy produced by these decays (per second per cubic meter) is small, but the energy cannot escape easily, so Earth's core is very hot. Thermal energy in Earth's core is transferred to Earth's surface and away from it through the processes of convection, conduction, and radiation.

10.5 | Fission

Learning Objectives

By the end of this section, you will be able to:

- Describe the process of nuclear fission in terms of its product and reactants
- Calculate the energies of particles produced by a fission reaction
- Explain the fission concept in the context of fission bombs and nuclear reactions

In 1934, Enrico Fermi bombarded chemical elements with neutrons in order to create isotopes of other elements. He assumed that bombarding uranium with neutrons would make it unstable and produce a new element. Unfortunately, Fermi could not determine the products of the reaction. Several years later, Otto Hahn and Fritz Strassman reproduced these experiments and discovered that the products of these reactions were smaller nuclei. From this, they concluded that the uranium nucleus had split into two smaller nuclei.

The splitting of a nucleus is called **fission**. Interestingly, U-235 fission does not always produce the same fragments. Example fission reactions include:

$$\begin{aligned}
{}^{1}_{0}\text{n} + {}^{235}_{92}\text{U} &\rightarrow {}^{141}_{56}\text{Ba} + {}^{92}_{36}\text{Kr} + 3\,{}^{1}_{0}\text{n} + Q, \\
{}^{1}_{0}\text{n} + {}^{235}_{92}\text{U} &\rightarrow {}^{140}_{54}\text{Xe} + {}^{94}_{38}\text{Sr} + 2\,{}^{1}_{0}\text{n} + Q, \\
{}^{1}_{0}\text{n} + {}^{235}_{92}\text{U} &\rightarrow {}^{132}_{50}\text{Sn} + {}^{101}_{42}\text{Mo} + 3\,{}^{1}_{0}\text{n} + Q.
\end{aligned}$$

In each case, the sum of the masses of the product nuclei are less than the masses of the reactants, so the fission of uranium is an exothermic process $(Q > 0)$. This is the idea behind the use of fission reactors as sources of energy (**Figure 10.17**).

The energy carried away by the reaction takes the form of particles with kinetic energy. The percent yield of fragments from a U-235 fission is given in **Figure 10.18**.

Figure 10.17 The Phillipsburg Nuclear Power Plant in Germany uses a fission reactor to generate electricity.

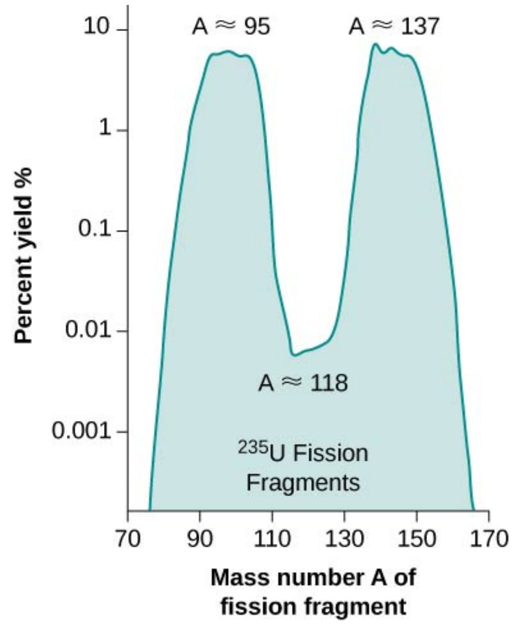

Figure 10.18 In this graph of fission fragments from U-235, the peaks in the graph indicate nuclei that are produced in the greatest abundance by the fission process.

Energy changes in a nuclear fission reaction can be understood in terms of the binding energy per nucleon curve (**Figure 10.7**). The BEN value for uranium $(A = 236)$ is slightly lower than its daughter nuclei, which lie closer to the iron (Fe) peak. This means that nucleons in the nuclear fragments are more tightly bound than those in the U-235 nucleus. Therefore, a fission reaction results in a drop in the average energy of a nucleon. This energy is carried away by high-energy neutrons.

Niels Bohr and John Wheeler developed the **liquid drop model** to understand the fission process. According to this model, firing a neutron at a nucleus is analogous to disturbing a droplet of water (**Figure 10.19**). The analogy works because short-range forces between nucleons in a nucleus are similar to the attractive forces between water molecules in a water droplet. In particular, forces between nucleons at the surface of the nucleus result in a surface tension similar to that of a water droplet. A neutron fired into a uranium nucleus can set the nucleus into vibration. If this vibration is violent enough, the nucleus divides into smaller nuclei and also emits two or three individual neutrons.

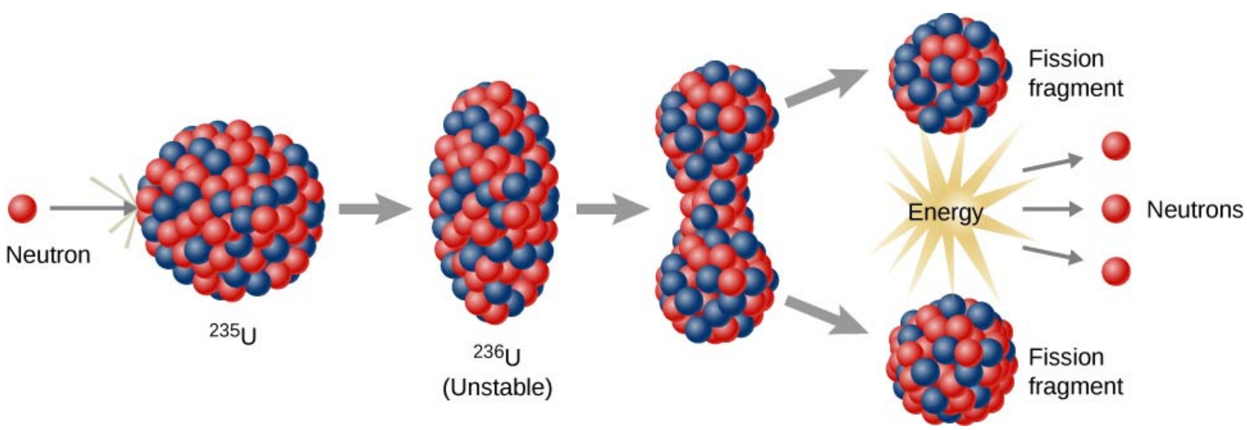

Figure 10.19 In the liquid drop model of nuclear fission, the uranium nucleus is split into two lighter nuclei by a high-energy neutron.

U-235 fission can produce a chain reaction. In a compound consisting of many U-235 nuclei, neutrons in the decay of one U-235 nucleus can initiate the fission of additional U-235 nuclei (**Figure 10.20**). This chain reaction can proceed in a controlled manner, as in a nuclear reactor at a power plant, or proceed uncontrollably, as in an explosion.

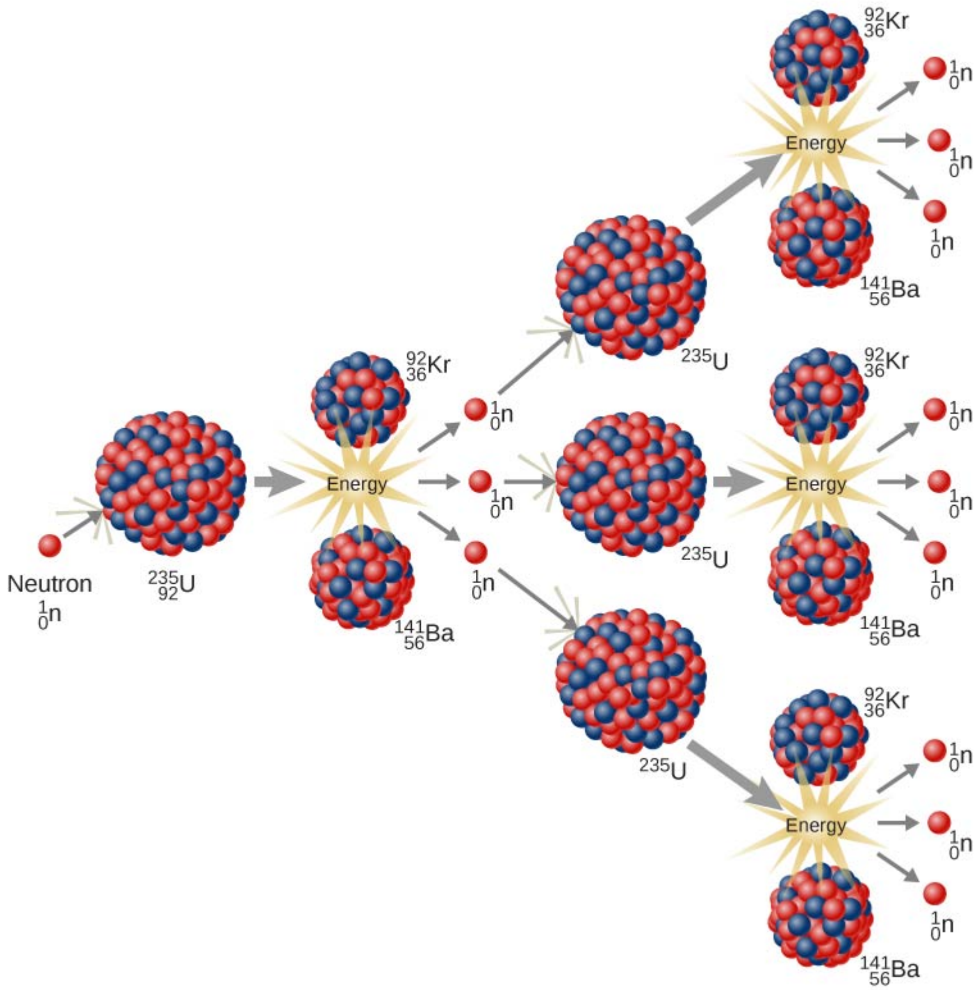

Figure 10.20 In a U-235 fission chain reaction, the fission of the uranium nucleus produces high-energy neutrons that go on to split more nuclei. The energy released in this process can be used to produce electricity.

View a simulation on **nuclear fission (https://openstaxcollege.org/l/21nuclrfissvid)** to start a chain reaction, or introduce nonradioactive isotopes to prevent one. Control energy production in a nuclear reactor.

The Atomic Bomb

The possibility of a chain reaction in uranium, with its extremely large energy release, led nuclear scientists to conceive of making a bomb—an atomic bomb. (These discoveries were taking place in the years just prior to the Second World War and many of the European physicists involved in these discoveries came from countries that were being overrun.) Natural uranium contains 99.3% U-238 and only 0.7% U-235, and does not produce a chain reaction. To produce a controlled, sustainable chain reaction, the percentage of U-235 must be increased to about 50% . In addition, the uranium sample must be massive enough so a typical neutron is more likely to induce fission than it is to escape. The minimum mass needed for the chain reaction to occur is called the **critical mass**. When the critical mass reaches a point at which the chain reaction becomes self-sustaining, this is a condition known as **criticality**. The original design required two pieces of U-235 below the critical mass. When one piece in the form of a bullet is fired into the second piece, the critical mass is exceeded and a chain reaction is produced.

An important obstacle to the U-235 bomb is the production of a critical mass of fissionable material. Therefore, scientists developed a plutonium-239 bomb because Pu-239 is more fissionable than U-235 and thus requires a smaller critical mass. The bomb was made in the form of a sphere with pieces of plutonium, each below the critical mass, at the edge of the sphere. A series of chemical explosions fired the plutonium pieces toward the center of the sphere simultaneously. When all these pieces of plutonium came together, the combination exceeded the critical mass and produced a chain reaction. Both the U-235 and Pu-239 bombs were used in World War II. Whether to develop and use atomic weapons remain two of the most important questions faced by human civilization.

Example 10.9

Calculating Energy Released by Fission

Calculate the energy released in the following spontaneous fission reaction:

$$^{238}\text{U} \rightarrow {}^{95}\text{Sr} + {}^{140}\text{Xe} + 3\text{n},$$

The atomic masses are $m\left(^{238}\text{U}\right) = 238.050784\text{ u}$, $m\left(^{95}\text{Sr}\right) = 94.919388\text{ u}$, $m\left(^{140}\text{Xe}\right) = 139.921610\text{ u}$, and $m(\text{n}) = 1.008665\text{ u}$.

Strategy

As always, the energy released is equal to the mass destroyed times c^2, so we must find the difference in mass between the parent ^{238}U and the fission products.

Solution

The products have a total mass of

$$\begin{aligned} m_{\text{products}} &= 94.919388\text{ u} + 139.921610\text{ u} + 3(1.008665\text{ u}) \\ &= 237.866993\text{ u}. \end{aligned}$$

The mass lost is the mass of $^{238}\text{U} - m_{\text{products}}$ or

$$\Delta m = 238.050784\text{ u} - 237.8669933\text{ u} = 0.183791\text{ u}.$$

Therefore, the energy released is

$$E = (\Delta m)c^2 = (0.183791\text{ u})\frac{931.5\text{ MeV}/c^2}{\text{u}}c^2 = 171.2\text{ MeV}.$$

Significance

Several important things arise in this example. The energy release is large but less than it would be if the nucleus split into two equal parts, since energy is carried away by neutrons. However, this fission reaction produces neutrons and does not split the nucleus into two equal parts. Fission of a given nuclide, such as ^{238}U, does not always produce the same products. Fission is a statistical process in which an entire range of products are produced with various probabilities. Most fission produces neutrons, although the number varies. This is an extremely important aspect of fission, because *neutrons can induce more fission*, enabling self-sustaining chain reactions.

Fission Nuclear Reactors

The first nuclear reactor was built by Enrico Fermi on a squash court on the campus of the University of Chicago on December 2, 1942. The reactor itself contained U-238 enriched with 3.6% U-235. Neutrons produced by the chain reaction move too fast to initiate fission reactions. One way to slow them down is to enclose the entire reactor in a water bath under high pressure. The neutrons collide with the water molecules and are slowed enough to be used in the fission process. The slowed neutrons split more U-235 nuclei and a chain reaction occurs. The rate at which the chain reaction proceeds is controlled by a series of “control” rods made of cadmium inserted into the reactor. Cadmium is capable of absorbing a large number of neutrons without becoming unstable.

A nuclear reactor design, called a pressurized water reactor, can also be used to generate electricity (**Figure 10.21**). A pressurized water reactor (on the left in the figure) is designed to control the fission of large amounts of ^{235}U. The energy released in this process is absorbed by water flowing through pipes in the system (the “primary loop”) and steam is produced. Cadmium control rods adjust the neutron flux (the rate of flow of neutrons passing through the system) and therefore control the reaction. In case the reactor overheats and the water boils away, the chain reaction terminates, because water is used to thermalize the neutrons. (This safety feature can be overwhelmed in extreme circumstances.) The hot, high-pressure water then passes through a pipe to a second tank of water at normal pressure in the steam generator. The steam produced at one end of the steam generator fills a chamber that contains a turbine. This steam is at a very high pressure. Meanwhile, a steam condenser connected to the other side of the turbine chamber maintains steam at low pressure. The pressure differences force steam through the chamber, which turns the turbine. The turbine, in turn, powers an electric generator.

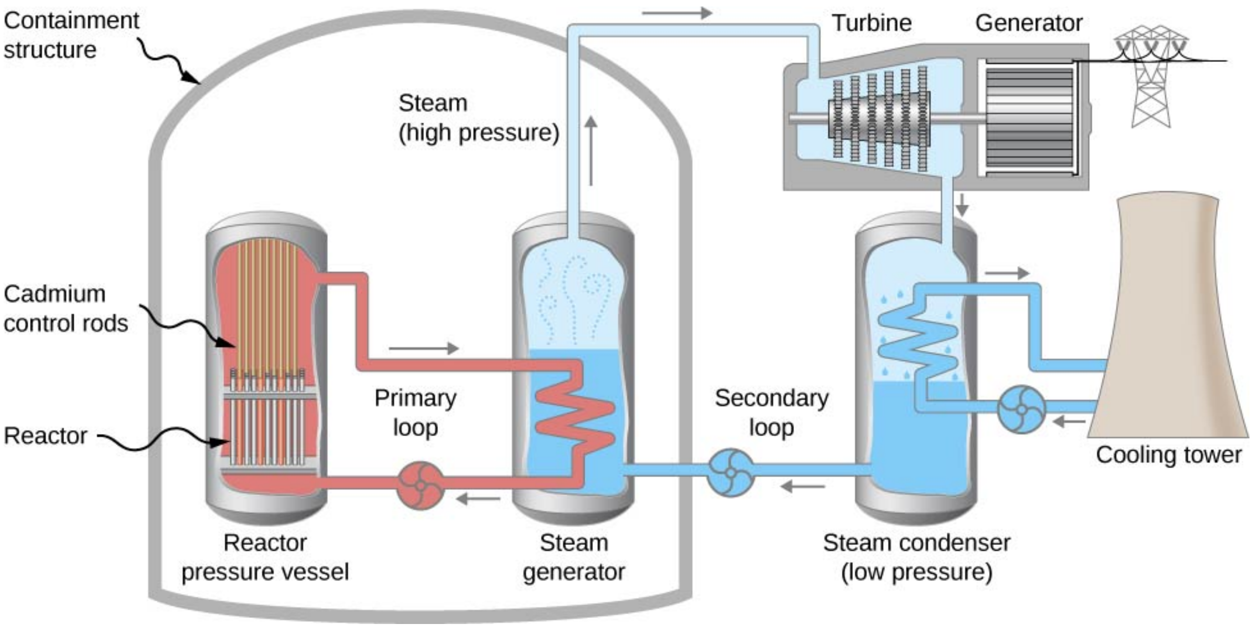

Figure 10.21 A nuclear reactor uses the energy produced in the fission of U-235 to produce electricity. Energy from a nuclear fission reaction produces hot, high-pressure steam that turns a turbine. As the turbine turns, electricity is produced.

The major drawback to a fission reactor is nuclear waste. U-235 fission produces nuclei with long half-lives such as ^{236}U that must be stored. These products cannot be dumped into oceans or left in any place where they will contaminate the environment, such as through the soil, air, or water. Many scientists believe that the best place to store nuclear waste is the bottom of old salt mines or inside of stable mountains.

Many people are fearful that a nuclear reactor may explode like an atomic bomb. However, a nuclear reactor does not contain enough U-235 to do this. Also, a nuclear reactor is designed so that failure of any mechanism of the reactor causes the cadmium control rods to fall fully into the reactor, stopping the fission process. As evidenced by the Fukushima and Chernobyl disasters, such systems can fail. Systems and procedures to avoid such disasters is an important priority for advocates of nuclear energy.

If all electrical power were produced by nuclear fission of U-235, Earth's known reserves of uranium would be depleted in less than a century. However, Earth's supply of fissionable material can be expanded considerably using a **breeder reactor**. A breeder reactor operates for the first time using the fission of U-235 as just described for the pressurized water reactor. But in addition to producing energy, some of the fast neutrons originating from the fission of U-235 are absorbed by U-238, resulting in the production of Pu-239 via the set of reactions

$$^{1}_{0}\text{n} + {}^{238}_{92}\text{U} \rightarrow {}^{239}_{92}\text{U} \xrightarrow{\beta} {}^{239}_{93}\text{Np} \xrightarrow{\beta} {}^{239}_{94}\text{Pu}.$$

The Pu-239 is itself highly fissionable and can therefore be used as a nuclear fuel in place of U-235. Since 99.3% of naturally occurring uranium is the U-238 isotope, the use of breeder reactors should increase our supply of nuclear fuel by roughly a factor of 100. Breeder reactors are now in operation in Great Britain, France, and Russia. Breeder reactors also have drawbacks. First, breeder reactors produce plutonium, which can, if leaked into the environment, produce serious public health problems. Second, plutonium can be used to build bombs, thus increasing significantly the risk of nuclear proliferation.

Example 10.10

Calculating Energy of Fissionable Fuel

Calculate the amount of energy produced by the fission of 1.00 kg of ^{235}U given that the average fission reaction of ^{235}U produces 200 MeV.

Strategy

The total energy produced is the number of ^{235}U atoms times the given energy per ^{235}U fission. We should therefore find the number of ^{235}U atoms in 1.00 kg.

Solution

The number of ^{235}U atoms in 1.00 kg is Avogadro's number times the number of moles. One mole of ^{235}U has a mass of 235.04 g; thus, there are $(1000\text{ g})/(235.04\text{ g/mol}) = 4.25\text{ mol}$. The number of ^{235}U atoms is therefore

$$(4.25\text{ mol})\left(6.02 \times 10^{23}\ ^{235}\text{U/mol}\right) = 2.56 \times 10^{24}\ ^{235}\text{U}.$$

Thus, the total energy released is

$$E = \left(2.56 \times 10^{24}\ ^{235}\text{U}\right)\left(\frac{200\text{ MeV}}{^{235}\text{U}}\right)\left(\frac{1.60 \times 10^{-13}\text{ J}}{\text{MeV}}\right) = 8.21 \times 10^{13}\text{ J}.$$

Significance

This is another impressively large amount of energy, equivalent to about 14,000 barrels of crude oil or 600,000 gallons of gasoline. However, it is only one-fourth the energy produced by the fusion of a kilogram mixture of deuterium and tritium. Even though each fission reaction yields about 10 times the energy of a fusion reaction, the energy per kilogram of fission fuel is less, because there are far fewer moles per kilogram of the heavy nuclides. Fission fuel is also much scarcer than fusion fuel, and less than 1% of uranium (the ^{235}U) is readily usable.

10.5 Check Your Understanding Which has a larger energy yield per fission reaction, a large or small sample of pure ^{235}U?

10.6 | Nuclear Fusion

Learning Objectives

By the end of this section, you will be able to:

- Describe the process of nuclear fusion in terms of its product and reactants
- Calculate the energies of particles produced by a fusion reaction
- Explain the fission concept in the context of fusion bombs, the production of energy by the Sun, and nucleosynthesis

The process of combining lighter nuclei to make heavier nuclei is called **nuclear fusion**. As with fission reactions, fusion reactions are exothermic—they release energy. Suppose that we fuse a carbon and helium nuclei to produce oxygen:

$$^{12}_{6}\text{C} + {}^{4}_{2}\text{He} \rightarrow {}^{16}_{8}\text{O} + \gamma.$$

The energy changes in this reaction can be understood using a graph of binding energy per nucleon (**Figure 10.7**). Comparing the binding energy per nucleon for oxygen, carbon, and helium, the oxygen nucleus is much more tightly bound than the carbon and helium nuclei, indicating that the reaction produces a drop in the energy of the system. This energy is released in the form of gamma radiation. Fusion reactions are said to be exothermic when the amount of energy released (known as the *Q value*) in each reaction is greater than zero $(Q > 0)$.

An important example of nuclear fusion in nature is the production of energy in the Sun. In 1938, Hans Bethe proposed that the Sun produces energy when hydrogen nuclei (^1H) fuse into stable helium nuclei $\left(^4\text{He}\right)$ in the Sun's core (**Figure 10.22**). This process, called the **proton-proton chain**, is summarized by three reactions:

$$
\begin{aligned}
&{}^1_1\text{H} + {}^1_1\text{H} \rightarrow {}^2_1\text{H} + {}^{\;\,0}_{+1}\text{e} + \nu + Q, \\
&{}^1_1\text{H} + {}^2_1\text{H} \rightarrow {}^3_2\text{He} + \gamma + Q, \\
&{}^3_2\text{He} + {}^3_2\text{He} \rightarrow {}^4_2\text{He} + {}^1_1\text{H} + {}^1_1\text{H} + Q.
\end{aligned}
$$

Thus, a stable helium nucleus is formed from the fusion of the nuclei of the hydrogen atom. These three reactions can be summarized by

$$4\,{}^1_1\text{H} \rightarrow {}^4_2\text{He} + 2\,{}^{\;\,0}_{+1}\text{e} + 2\gamma + 2\nu + Q.$$

The net *Q* value is about 26 MeV. The release of this energy produces an outward thermal gas pressure that prevents the Sun from gravitational collapse. Astrophysicists find that hydrogen fusion supplies the energy stars require to maintain energy balance over most of a star's life span.

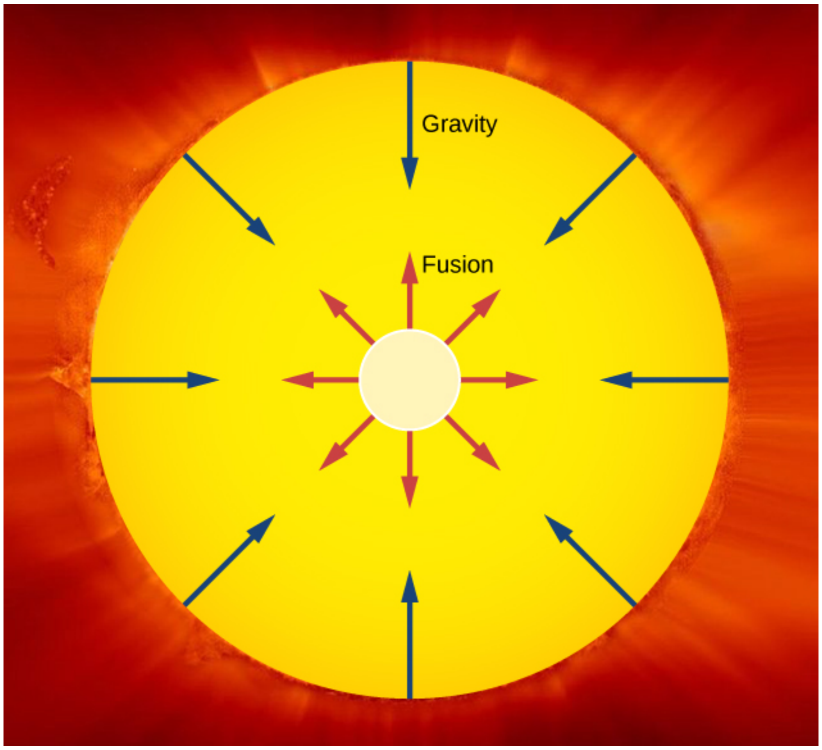

Figure 10.22 The Sun produces energy by fusing hydrogen into helium at the Sun's core. The red arrows show outward pressure due to thermal gas, which tends to make the Sun expand. The blue arrows show inward pressure due to gravity, which tends to make the Sun contract. These two influences balance each other.

Nucleosynthesis

Scientist now believe that many heavy elements found on Earth and throughout the universe were originally synthesized by fusion within the hot cores of the stars. This process is known as **nucleosynthesis**. For example, in lighter stars, hydrogen combines to form helium through the proton-proton chain. Once the hydrogen fuel is exhausted, the star enters the next stage of its life and fuses helium. An example of a nuclear reaction chain that can occur is:

$$\begin{aligned} {}^{4}_{2}\mathrm{He} + {}^{4}_{2}\mathrm{He} &\rightarrow {}^{8}_{4}\mathrm{Be} + \gamma, \\ {}^{8}_{4}\mathrm{Be} + {}^{4}_{2}\mathrm{He} &\rightarrow {}^{12}_{6}\mathrm{C} + \gamma, \\ {}^{12}_{6}\mathrm{C} + {}^{4}_{2}\mathrm{He} &\rightarrow {}^{16}_{8}\mathrm{O} + \gamma. \end{aligned}$$

Carbon and oxygen nuclei produced in such processes eventually reach the star's surface by convection. Near the end of its lifetime, the star loses its outer layers into space, thus enriching the interstellar medium with the nuclei of heavier elements (**Figure 10.23**).

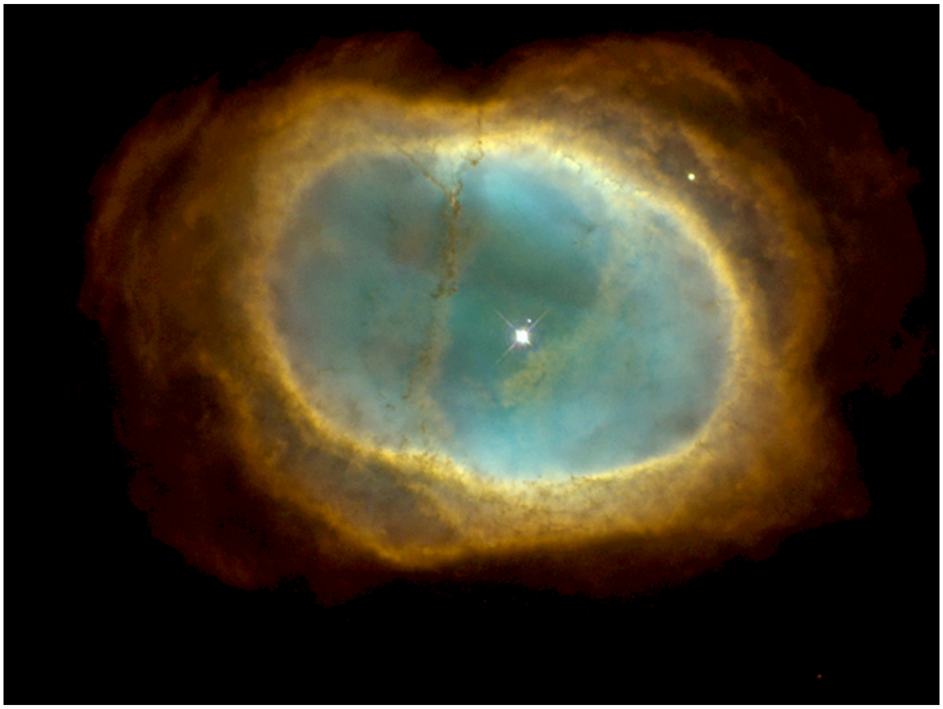

Figure 10.23 A planetary nebula is produced at the end of the life of a star. The greenish color of this planetary nebula comes from oxygen ions. (credit: Hubble Heritage Team (STScI/AURA/NASA/ESA))

Stars similar in mass to the Sun do not become hot enough to fuse nuclei as heavy (or heavier) than oxygen nuclei. However, in massive stars whose cores become much hotter $\left(T > 6 \times 10^{8}\ \mathrm{K}\right),$ even more complex nuclei are produced. Some representative reactions are

$$\begin{aligned} {}^{12}_{6}\mathrm{C} + {}^{12}_{6}\mathrm{C} &\rightarrow {}^{23}_{11}\mathrm{Na} + {}^{1}_{1}\mathrm{H}, \\ {}^{12}_{6}\mathrm{C} \;\rightarrow {}^{12}_{6}\mathrm{C} &\rightarrow {}^{24}_{12}\mathrm{Mg} + \gamma, \\ {}^{12}_{6}\mathrm{C} + {}^{16}_{8}\mathrm{O} &\rightarrow {}^{28}_{14}\mathrm{Si} + \gamma. \end{aligned}$$

Nucleosynthesis continues until the core is primarily iron-nickel metal. Now, iron has the peculiar property that any fusion or fission reaction involving the iron nucleus is endothermic, meaning that energy is absorbed rather than produced. Hence, nuclear energy cannot be generated in an iron-rich core. Lacking an outward pressure from fusion reactions, the star begins to contract due to gravity. This process heats the core to a temperature on the order of $5 \times 10^{9}\,\mathrm{K}.$ Expanding shock waves generated within the star due to the collapse cause the star to quickly explode. The luminosity of the star can increase temporarily to nearly that of an entire galaxy. During this event, the flood of energetic neutrons reacts with iron and the other nuclei to produce elements heavier than iron. These elements, along with much of the star, are ejected into space by the explosion. Supernovae and the formation of planetary nebulas together play a major role in the dispersal of chemical elements into space.

Eventually, much of the material lost by stars is pulled together through the gravitational force, and it condenses into a new generation of stars and accompanying planets. Recent images from the Hubble Space Telescope provide a glimpse of this magnificent process taking place in the constellation Serpens (**Figure 10.24**). The new generation of stars begins the nucleosynthesis process anew, with a higher percentage of heavier elements. Thus, stars are “factories” for the chemical elements, and many of the atoms in our bodies were once a part of stars.

Figure 10.24 This image taken by NASA’s Spitzer Space Telescope and the Two Micron All Sky Survey (2MASS), shows the Serpens Cloud Core, a star-forming region in the constellation Serpens (the “Serpent”). Located about 750 light-years away, this cluster of stars is formed from cooling dust and gases. Infrared light has been used to reveal the youngest stars in orange and yellow. (credit: NASA/JPL-Caltech/2MASS)

Example 10.11

Energy of the Sun

The power output of the Sun is approximately 3.8×10^{26} J/s. Most of this energy is produced in the Sun’s core by the proton-proton chain. This energy is transmitted outward by the processes of convection and radiation. (a) How many of these fusion reactions per second must occur to supply the power radiated by the Sun? (b) What is the rate at which the mass of the Sun decreases? (c) In about five billion years, the central core of the Sun will be depleted of hydrogen. By what percentage will the mass of the Sun have decreased from its present value when the core is depleted of hydrogen?

Strategy

The total energy output per second is given in the problem statement. If we know the energy released in each fusion reaction, we can determine the rate of the fusion reactions. If the mass loss per fusion reaction is known, the mass loss rate is known. Multiplying this rate by five billion years gives the total mass lost by the Sun. This value is divided by the original mass of the Sun to determine the percentage of the Sun’s mass that has been lost when the hydrogen fuel is depleted.

Solution

a. The decrease in mass for the fusion reaction is

$$\begin{aligned}\Delta m &= 4m\left({}^1_1\text{H}\right) - m\left({}^4_2\text{He}\right) - 2m\left({}^{\;\;0}_{+1}\text{e}\right)\\ &= 4(1.007825\text{ u}) - 4.002603\text{ u} - 2(0.000549\text{ u})\\ &= 0.0276\text{ u}.\end{aligned}$$

The energy released per fusion reaction is

$$Q = (0.0276\text{ u})(931.49\text{ MeV/u}) = 25.7\text{ MeV}.$$

Thus, to supply $3.8 \times 10^{26}\text{ J/s} = 2.38 \times 10^{39}\text{ MeV/s},$ there must be

$$\frac{2.38 \times 10^{39}\text{ MeV/s}}{25.7\text{ MeV/reaction}} = 9.26 \times 10^{37}\text{ reaction/s}.$$

b. The Sun's mass decreases by $0.0276\,\text{u} = 4.58 \times 10^{-29}\,\text{kg}$ per fusion reaction, so the rate at which its mass decreases is

$$\left(9.26 \times 10^{37}\ \text{reaction/s}\right)\left(4.58 \times 10^{-29}\ \text{kg/reaction}\right) = 4.24 \times 10^{9}\ \text{kg/s}.$$

c. In $5 \times 10^{9}\ \text{y} = 1.6 \times 10^{17}\ \text{s},$ the Sun's mass will therefore decrease by

$$\Delta M = \left(4.24 \times 10^{9}\ \text{kg/s}\right)\left(1.6 \times 10^{17}\ \text{s}\right) = 6.8 \times 10^{26}\ \text{kg}.$$

The current mass of the Sun is about $2.0 \times 10^{30}\ \text{kg},$ so the percentage decrease in its mass when its hydrogen fuel is depleted will be

$$\left(\frac{6.8 \times 10^{26}\ \text{kg}}{2.0 \times 10^{30}\ \text{kg}}\right) \times 100\% = 0.034\%.$$

Significance

After five billion years, the Sun is very nearly the same mass as it is now. Hydrogen burning does very little to change the mass of the Sun. This calculation assumes that only the proton-proton decay change is responsible for the power output of the Sun.

10.6 Check Your Understanding Where does the energy from the Sun originate?

The Hydrogen Bomb

In 1942, Robert Oppenheimer suggested that the extremely high temperature of an atomic bomb could be used to trigger a fusion reaction between deuterium and tritium, thus producing a fusion (or hydrogen) bomb. The reaction between deuterium and tritium, both isotopes of hydrogen, is given by

$${}_{1}^{2}\text{H} + {}_{1}^{3}\text{H} \rightarrow {}_{2}^{4}\text{He} + {}_{0}^{1}\text{n} + 17.6\,\text{MeV}.$$

Deuterium is relatively abundant in ocean water but tritium is scarce. However, tritium can be generated in a nuclear reactor through a reaction involving lithium. The neutrons from the reactor cause the reaction

$${}_{0}^{1}\text{n} + {}_{3}^{7}\text{Li} \rightarrow {}_{2}^{4}\text{He} + {}_{1}^{3}\text{H} + {}_{0}^{1}\text{n},$$

to produce the desired tritium. The first hydrogen bomb was detonated in 1952 on the remote island of Eniwetok in the Marshall Islands. A hydrogen bomb has never been used in war. Modern hydrogen bombs are approximately 1000 times more powerful than the fission bombs dropped on Hiroshima and Nagasaki in World War II.

The Fusion Reactor

The fusion chain believed to be the most practical for use in a **nuclear fusion reactor** is the following two-step process:

$${}_{1}^{2}\text{H} + {}_{1}^{2}\text{H} \rightarrow {}_{1}^{3}\text{H} + {}_{1}^{1}\text{H},$$
$${}_{1}^{2}\text{H} + {}_{1}^{3}\text{H} \rightarrow {}_{2}^{4}\text{He} + {}_{0}^{1}\text{n}.$$

This chain, like the proton-proton chain, produces energy without any radioactive by-product. However, there is a very difficult problem that must be overcome before fusion can be used to produce significant amounts of energy: Extremely high temperatures $\left(\sim 10^{7}\,\text{K}\right)$ are needed to drive the fusion process. To meet this challenge, test fusion reactors are being developed to withstand temperatures 20 times greater than the Sun's core temperature. An example is the Joint European Torus (JET) shown in Figure 10.25. A great deal of work still has to be done on fusion reactor technology, but many scientists predict that fusion energy will power the world's cities by the end of the twentieth century.

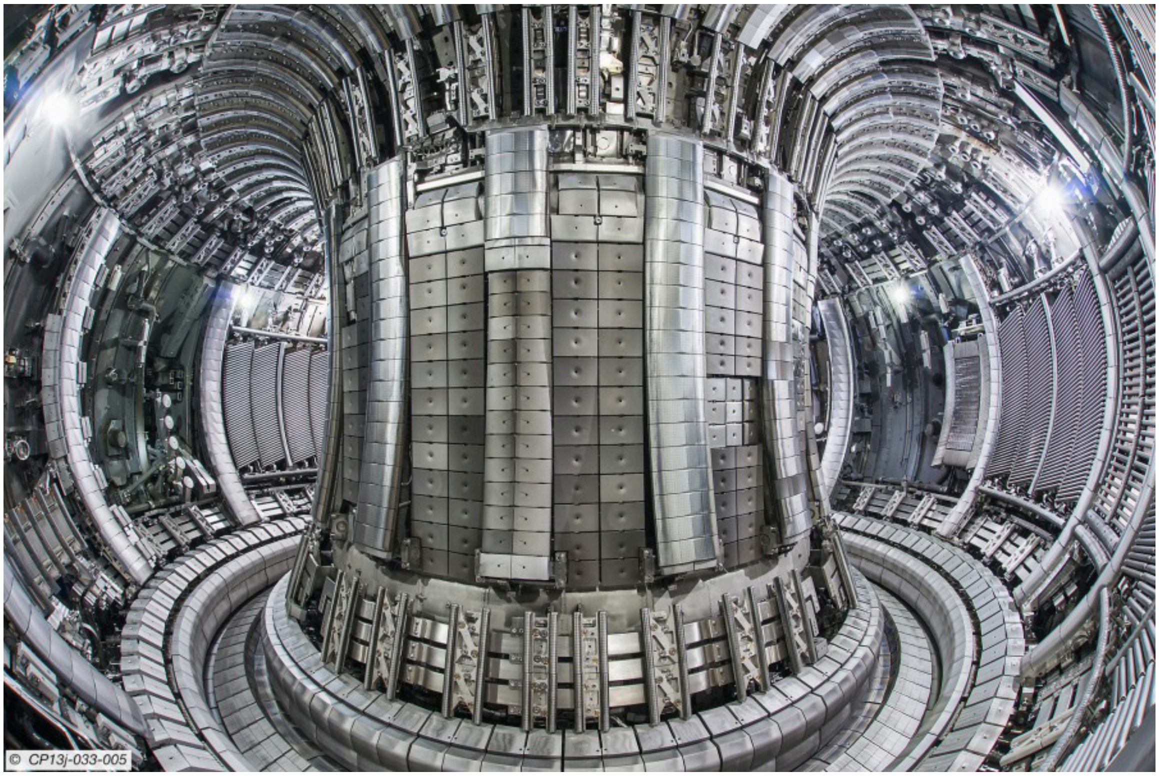

Figure 10.25 The Joint European Torus (JET) tokamak fusion detector uses magnetic fields to fuse deuterium and tritium nuclei (credit: EUROfusion).

10.7 | Medical Applications and Biological Effects of Nuclear Radiation

Learning Objectives

By the end of this section, you will be able to:

- Describe two medical uses of nuclear technology
- Explain the origin of biological effects due to nuclear radiation
- List common sources of radiation and their effects
- Estimate exposure for nuclear radiation using common dosage units

Nuclear physics is an integral part of our everyday lives (**Figure 10.26**). Radioactive compounds are used in to identify cancer, study ancient artifacts, and power our cities. Nuclear fusion also powers the Sun, the primary source of energy on Earth. The focus of this chapter is nuclear radiation. In this section, we ask such questions as: How is nuclear radiation used to benefit society? What are its health risks? How much nuclear radiation is the average person exposed to in a lifetime?

Figure 10.26 Dr. Tori Randall, a curator at the San Diego Museum of Man, uses nuclear radiation to study a 500-year-old Peruvian child mummy. The origin of this radiation is the transformation of one nucleus to another. (credit: Samantha A. Lewis, U.S. Navy)

Medical Applications

Medical use of nuclear radiation is quite common in today's hospitals and clinics. One of the most important uses of nuclear radiation is the location and study of diseased tissue. This application requires a special drug called a **radiopharmaceutical**. A radiopharmaceutical contains an unstable radioactive isotope. When the drug enters the body, it tends to concentrate in inflamed regions of the body. (Recall that the interaction of the drug with the body does not depend on whether a given nucleus is replaced by one of its isotopes, since this interaction is determined by chemical interactions.) Radiation detectors used outside the body use nuclear radiation from the radioisotopes to locate the diseased tissue. Radiopharmaceuticals are called **radioactive tags** because they allow doctors to track the movement of drugs in the body. Radioactive tags are for many purposes, including the identification of cancer cells in the bones, brain tumors, and Alzheimer's disease (**Figure 10.27**). Radioactive tags are also used to monitor the function of body organs, such as blood flow, heart muscle activity, and iodine uptake in the thyroid gland.

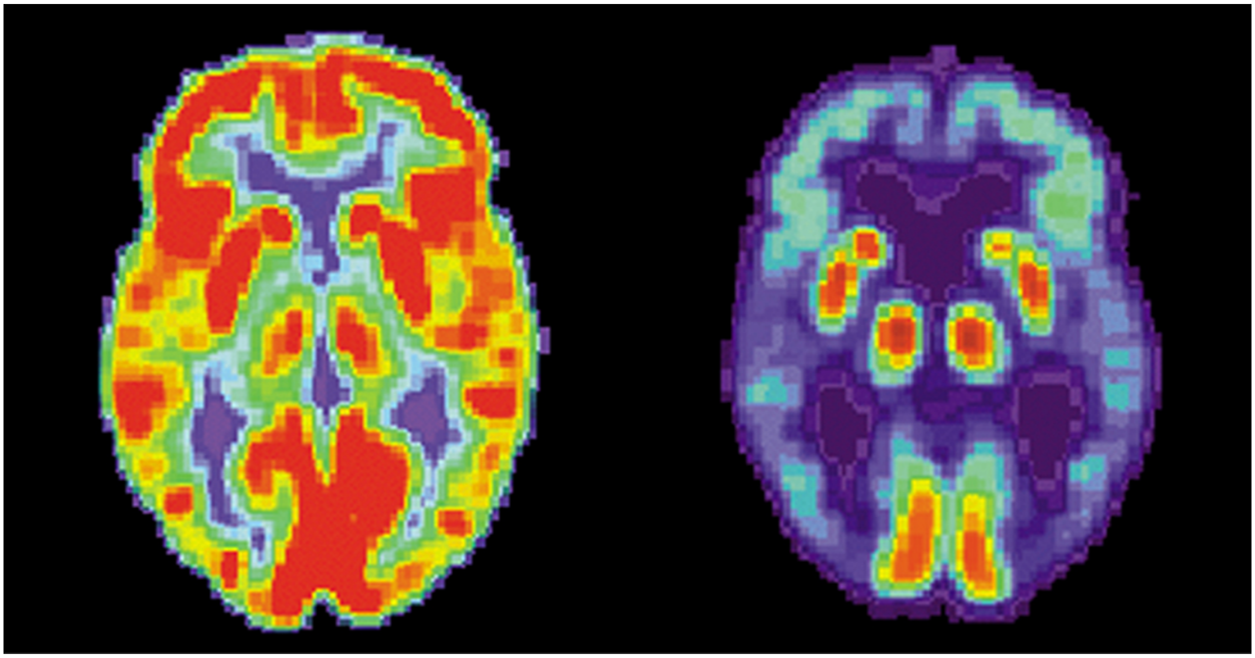

Figure 10.27 These brain images are produced using a radiopharmaceutical. The colors indicate relative metabolic or biochemical activity (red indicates high activity and blue indicates low activity). The figure on the left shows the normal brain of an individual and the figure on the right shows the brain of someone diagnosed with Alzheimer's disease. The brain image of the normal brain indicates much greater metabolic activity (a larger fraction of red and orange areas). (credit: modification of works by National Institutes of Health)

Table 10.2 lists some medical diagnostic uses of radiopharmaceuticals, including isotopes and typical activity (*A*) levels. One common diagnostic test uses iodine to image the thyroid, since iodine is concentrated in that organ. Another common nuclear diagnostic is the thallium scan for the cardiovascular system, which reveals blockages in the coronary arteries and examines heart activity. The salt TlCl can be used because it acts like NaCl and follows the blood. Note that **Table 10.2** lists many diagnostic uses for ^{99m}Tc, where "m" stands for a metastable state of the technetium nucleus. This isotope is used in many compounds to image the skeleton, heart, lungs, and kidneys. About 80% of all radiopharmaceuticals employ ^{99m}Tc because it produces a single, easily identified, 0.142-MeV γ ray and has a short 6.0-h half-life, which reduces radiation exposure.

Procedure, Isotope	Activity (mCi), where $1\ \text{mCi} = 3.7 \times 10^7\ \text{Bq}$	Procedure, Isotope	Activity (mCi), where $1\ \text{mCi} = 3.7 \times 10^7\ \text{Bq}$
Brain scan		*Thyroid scan*	
^{99m}Tc	7.5	^{131}I	0.05
^{15}O (PET)	50	^{123}I	0.07
Lung scan		*Liver scan*	
^{13}Xe	7.5	^{198}Au (colloid)	0.1
^{99m}Tc	2	^{99m}Tc (colloid)	2
Cardiovascular blood pool		*Bone scan*	

Table 10.2 Diagnostic Uses of Radiopharmaceuticals

Procedure, Isotope	Activity (mCi), where $1\ \text{mCi} = 3.7 \times 10^7\ \text{Bq}$	Procedure, Isotope	Activity (mCi), where $1\ \text{mCi} = 3.7 \times 10^7\ \text{Bq}$
^{131}I	0.2	^{85}Sr	0.1
$^{99\text{m}}\text{Tc}$	2	$^{99\text{m}}\text{Tc}$	10
Cardiovascular arterial flow		*Kidney scan*	
^{201}Tl	3	^{197}Hg	0.1
^{24}Na	7.5	$^{99\text{m}}\text{Tc}$	1.5

Table 10.2 Diagnostic Uses of Radiopharmaceuticals

The first radiation detectors produced two-dimensional images, like a photo taken from a camera. However, a circular array of detectors that can be rotated can be used to produce three-dimensional images. This technique is similar to that used in X-ray computed tomography (CT) scans. One application of this technique is called **single-photon-emission CT (SPECT)** (Figure 10.28). The spatial resolution of this technique is about 1 cm.

Figure 10.28 The SPECT machine uses radiopharmaceutical compounds to produce an image of the human body. The machine takes advantage of the physics of nuclear beat decays and electron-positron collisions. (credit: “Woldo”/Wikimedia Commons)

Improved image resolution is achieved by a technique known as **positron emission tomography (PET)**. This technique use radioisotopes that decay by β^+ radiation. When a positron encounters an electron, these particle annihilate to produce two gamma-ray photons. This reaction is represented by

$$e^+ + e^- \rightarrow 2\gamma.$$

These γ-ray photons have identical 0.511-MeV energies and move directly away from one another (Figure 10.29). This easily identified decay signature can be used to identify the location of the radioactive isotope. Examples of β^+-emitting isotopes used in PET include ^{11}C, ^{13}N, ^{15}O, and ^{18}F. The nuclei have the advantage of being able to function as tags for natural body compounds. Its resolution of 0.5 cm is better than that of SPECT.

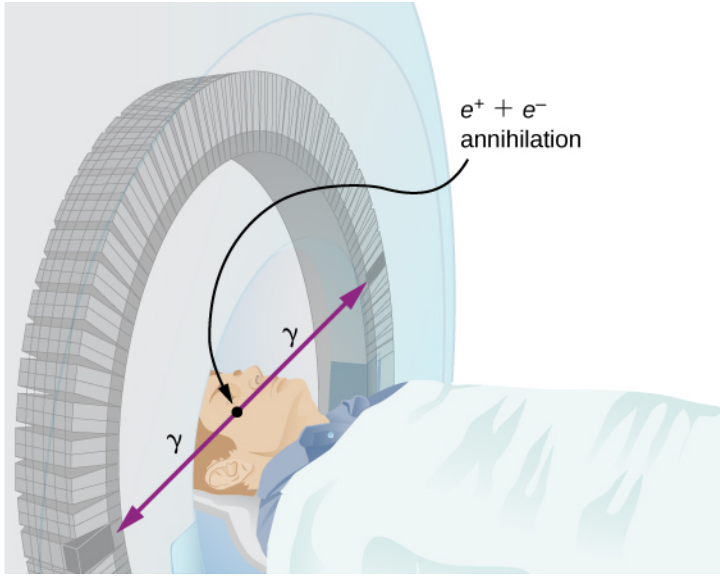

Figure 10.29 A PET system takes advantage of the two identical γ -ray photons produced by positron-electron annihilation. These γ rays are emitted in opposite directions, so that the line along which each pair is emitted is determined.

PET scans are especially useful to examine the brain's anatomy and function. For example, PET scans can be used to monitor the brain's use of oxygen and water, identify regions of decreased metabolism (linked to Alzheimer's disease), and locate different parts of the brain responsible for sight, speech, and fine motor activity

Is it a tumor? View an **animation (https://openstaxcollege.org/l/21simmagresimg)** of simplified magnetic resonance imaging (MRI) to see if you can tell. Your head is full of tiny radio transmitters (the nuclear spins of the hydrogen nuclei of your water molecules). In an MRI unit, these little radios can be made to broadcast their positions, giving a detailed picture of the inside of your head.

Biological Effects

Nuclear radiation can have both positive and negative effects on biological systems. However, it can also be used to treat and even cure cancer. How do we understand these effects? To answer this question, consider molecules within cells, particularly DNA molecules.

Cells have long, double-helical DNA molecules containing chemical codes that govern the function and processes of the cell. Nuclear radiation can alter the structural features of the DNA chain, leading to changes in the genetic code. In human cells, we can have as many as a million individual instances of damage to DNA per cell per day. DNA contains codes that check whether the DNA is damaged and can repair itself. This repair ability of DNA is vital for maintaining the integrity of the genetic code and for the normal functioning of the entire organism. It should be constantly active and needs to respond rapidly. The rate of DNA repair depends on various factors such as the type and age of the cell. If nuclear radiation damages the ability of the cell to repair DNA, the cell can

1. Retreat to an irreversible state of dormancy (known as senescence);
2. Commit suicide (known as programmed cell death); or
3. Progress into unregulated cell division, possibly leading to tumors and cancers.

Nuclear radiation can harm the human body is many other ways as well. For example, high doses of nuclear radiation can cause burns and even hair loss.

Biological effects of nuclear radiation are expressed by many different physical quantities and in many different units. A common unit to express the biological effects of nuclear radiation is the **rad** or **radiation dose unit**. One rad is equal to 1/100 of a joule of nuclear energy deposited per kilogram of tissue, written:

$$1\text{ rad} = 0.01\text{ J/kg}.$$

For example, if a 50.0-kg person is exposed to nuclear radiation over her entire body and she absorbs 1.00 J, then her whole-body radiation dose is

$$(1.00\text{ J})/(50.0\text{ kg}) = 0.0200\text{ J/kg} = 2.00\text{ rad}.$$

Nuclear radiation damages cells by ionizing atoms in the cells as they pass through the cells (**Figure 10.30**). The effects of ionizing radiation depend on the dose in rads, but also on the type of radiation (alpha, beta, gamma, or X-ray) and the type of tissue. For example, if the range of the radiation is small, as it is for α rays, then the ionization and the damage created is more concentrated and harder for the organism to repair. To account for such affects, we define the **relative biological effectiveness** (RBE). Sample RBE values for several types of ionizing nuclear radiation are given in **Table 10.3**.

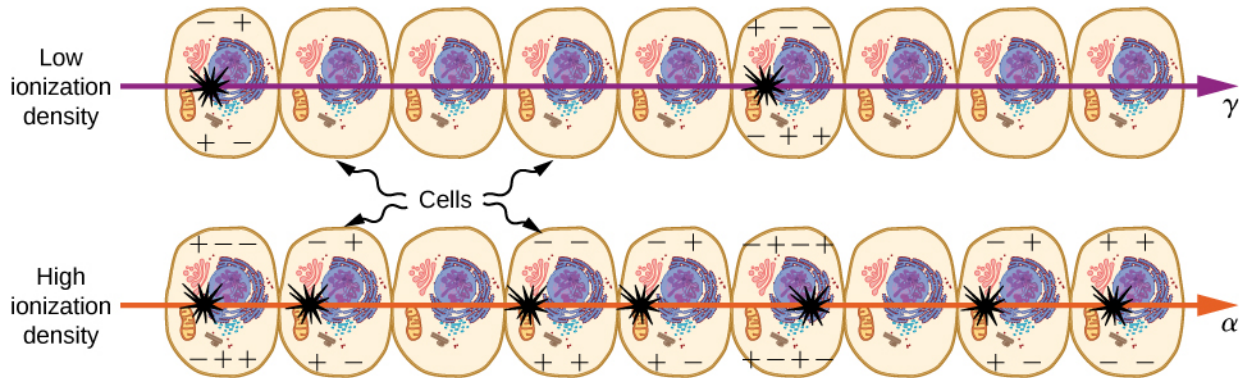

Figure 10.30 The image shows ionization created in cells by α and γ radiation. Because of its shorter range, the ionization and damage created by α rays is more concentrated and harder for the organism to repair. Thus, the RBE for α rays is greater than the RBE for γ rays, even though they create the same amount of ionization at the same energy.

Type and Energy of Radiation	RBE[1]
X-rays	1
γ rays	1
β rays greater than 32 keV	1
β rays less than 32 keV	1.7
Neutrons, thermal to slow (<20 keV)	2–5
Neutrons, fast (1–10 MeV)	10 (body), 32 (eyes)
Protons (1–10 MeV)	10 (body), 32 (eyes)
α rays from radioactive decay	10–20
Heavy ions from accelerators	10–20

Table 10.3 Relative Biological Effectiveness [1] Values approximate. Difficult to determine.

A dose unit more closely related to effects in biological tissue is called the **roentgen equivalent man (rem)** and is defined to be the dose (in rads) multiplied by the relative biological effectiveness (RBE). Thus, if a person had a whole-body dose of 2.00 rad of γ radiation, the dose in rem would be $(2.00\text{ rad})(1) = 2.00$ rem for the whole body. If the person had a whole-body dose of 2.00 rad of α radiation, then the dose in rem would be $(2.00\text{ rad})(20) = 40.0$ rem for the whole body. The α rays would have 20 times the effect on the person than the γ rays for the same deposited energy. The SI equivalent of the rem, and the more standard term, is the **sievert (Sv)** is

$$1\text{ Sv} = 100\text{ rem}.$$

The RBEs given in **Table 10.3** are approximate but reflect an understanding of nuclear radiation and its interaction with living tissue. For example, neutrons are known to cause more damage than γ rays, although both are neutral and have large ranges, due to secondary radiation. Any dose less than 100 mSv (10 rem) is called a **low dose**, 0.1 Sv to 1 Sv (10 to 100 rem) is called a **moderate dose**, and anything greater than 1 Sv (100 rem) is called a **high dose**. It is difficult to determine if a person has been exposed to less than 10 mSv.

Biological effects of different levels of nuclear radiation on the human body are given in **Table 10.4**. The first clue that a person has been exposed to radiation is a change in blood count, which is not surprising since blood cells are the most rapidly reproducing cells in the body. At higher doses, nausea and hair loss are observed, which may be due to interference with cell reproduction. Cells in the lining of the digestive system also rapidly reproduce, and their destruction causes nausea. When the growth of hair cells slows, the hair follicles become thin and break off. High doses cause significant cell death in all systems, but the lowest doses that cause fatalities do so by weakening the immune system through the loss of white blood cells.

Dose in Sv[1]	Effect
0–0.10	No observable effect.
0.1–1	Slight to moderate decrease in white blood cell counts.
0.5	Temporary sterility; 0.35 for women, 0.50 for men.
1–2	Significant reduction in blood cell counts, brief nausea and vomiting. Rarely fatal.
2–5	Nausea, vomiting, hair loss, severe blood damage, hemorrhage, fatalities.
4.5	Lethal to 50% of the population within 32 days after exposure if not treated.
5–20	Worst effects due to malfunction of small intestine and blood systems. Limited survival.
>20	Fatal within hours due to collapse of central nervous system.

Table 10.4 Immediate Effects of Radiation (Adults, Whole-Body, Single Exposure) [1] Multiply by 100 to obtain dose in rem.

Sources of Radiation

Human are also exposed to many sources of nuclear radiation. A summary of average radiation doses for different sources by country is given in **Table 10.5**. Earth emits radiation due to the isotopes of uranium, thorium, and potassium. Radiation levels from these sources depend on location and can vary by a factor of 10. Fertilizers contain isotopes of potassium and uranium, which we digest in the food we eat. Fertilizers have more than 3000 Bq/kg radioactivity, compared to just 66 Bq/kg for Carbon-14.

Source	Dose (mSv/y)[1]			
	Australia	Germany	US	World
Natural radiation – external				
Cosmic rays	0.30	0.28	0.30	0.39
Soil, building materials	0.40	0.40	0.30	0.48
Radon gas	0.90	1.1	2.0	1.2
Natural radiation – internal				
$^{40}K,\ ^{14}C,\ ^{226}Ra$	0.24	0.28	0.40	0.29
Artificial radiation				
Medical and dental	0.80	0.90	0.53	0.40
TOTAL	2.6	3.0	3.5	2.8

Table 10.5 Background Radiation Sources and Average Doses [1] Multiply by 100 to obtain does in mrem/y.

Medical visits are also a source of nuclear radiation. A sample of common nuclear radiation doses is given in **Table 10.6**.

These doses are generally low and can be lowered further with improved techniques and more sensitive detectors. With the possible exception of routine dental X-rays, medical use of nuclear radiation is used only when the risk-benefit is favorable. Chest X-rays give the lowest doses—about 0.1 mSv to the tissue affected, with less than 5% scattering into tissues that are not directly imaged. Other X-ray procedures range upward to about 10 mSv in a CT scan, and about 5 mSv (0.5 rem) per dental X-ray, again both only affecting the tissue imaged. Medical images with radiopharmaceuticals give doses ranging from 1 to 5 mSv, usually localized.

Procedure	Effective Dose (mSv)
Chest	0.02
Dental	0.01
Skull	0.07
Leg	0.02
Mammogram	0.40
Barium enema	7.0
Upper GI	3.0
CT head	2.0
CT abdomen	10.0

Table 10.6 Typical Doses Received During Diagnostic X-Ray Exams

Example 10.12

What Mass of ^{137}Cs Escaped Chernobyl?

The Chernobyl accident in Ukraine (formerly in the Soviet Union) exposed the surrounding population to a large amount of radiation through the decay of ^{137}Cs. The initial radioactivity level was approximately $A = 6.0\,\text{MCi}$. Calculate the total mass of ^{137}Cs involved in this accident.

Strategy

The total number of nuclei, *N*, can be determined from the known half-life and activity of ^{137}Cs (30.2 y). The mass can be calculated from *N* using the concept of a mole.

Solution

Solving the equation $A = \frac{0.693\,N}{t_{1/2}}$ for *N* gives

$$N = \frac{A\,t_{1/2}}{0.693}.$$

Entering the given values yields

$$N = \frac{(6.0\,\text{MCi})(30.2\,\text{y})}{0.693}.$$

To convert from curies to becquerels and years to seconds, we write

$$N = \frac{\left(6.0 \times 10^6\ \text{Ci}\right)\left(3.7 \times 10^{10}\ \text{Bq/Ci}\right)(30.2\,\text{y})\left(3.16 \times 10^7\ \text{s/y}\right)}{0.693} = 3.1 \times 10^{26}.$$

One mole of a nuclide ^{A}X has a mass of *A* grams, so that one mole of ^{137}Cs has a mass of 137 g. A mole has 6.02×10^{23} nuclei. Thus the mass of ^{137}Cs released was

$$m = \left(\frac{137\text{ g}}{6.02 \times 10^{23}}\right)\left(3.1 \times 10^{26}\right) = 70 \times 10^{3}\text{ g} = 70\text{ kg}.$$

Significance

The mass of ^{137}Cs involved in the Chernobyl accident is a small material compared to the typical amount of fuel used in a nuclear reactor. However, approximately 250 people were admitted to local hospitals immediately after the accident, and diagnosed as suffering acute radiation syndrome. They received external radiation dosages between 1 and 16 Sv. Referring to biological effects in **Table 10.4**, these dosages are extremely hazardous. The eventual death toll is estimated to be around 4000 people, primarily due to radiation-induced cancer.

10.7 Check Your Understanding Radiation propagates in all directions from its source, much as electromagnetic radiation from a light bulb. Is *activity* concept more analogous to power, intensity, or brightness?

CHAPTER 10 REVIEW

KEY TERMS

activity magnitude of the decay rate for radioactive nuclides

alpha (α) rays one of the types of rays emitted from the nucleus of an atom as alpha particles

alpha decay radioactive nuclear decay associated with the emission of an alpha particle

antielectrons another term for positrons

antineutrino antiparticle of an electron's neutrino in β^- decay

atomic mass total mass of the protons, neutrons, and electrons in a single atom

atomic mass unit unit used to express the mass of an individual nucleus, where $1\text{u} = 1.66054 \times 10^{-27}$ kg

atomic nucleus tightly packed group of nucleons at the center of an atom

atomic number number of protons in a nucleus

becquerel (Bq) SI unit for the decay rate of a radioactive material, equal to 1 decay/second

beta (β) rays one of the types of rays emitted from the nucleus of an atom as beta particles

beta decay radioactive nuclear decay associated with the emission of a beta particle

binding energy (BE) energy needed to break a nucleus into its constituent protons and neutrons

binding energy per nucleon (BEN) energy need to remove a nucleon from a nucleus

breeder reactor reactor that is designed to make plutonium

carbon-14 dating method to determine the age of formerly living tissue using the ratio $^{14}\text{C}/^{12}\text{C}$

chart of the nuclides graph comprising stable and unstable nuclei

critical mass minimum mass required of a given nuclide in order for self-sustained fission to occur

criticality condition in which a chain reaction easily becomes self-sustaining

curie (Ci) unit of decay rate, or the activity of 1 g of ^{226}Ra, equal to 3.70×10^{10} Bq

daughter nucleus nucleus produced by the decay of a parent nucleus

decay process by which an individual atomic nucleus of an unstable atom loses mass and energy by emitting ionizing particles

decay constant quantity that is inversely proportional to the half-life and that is used in equation for number of nuclei as a function of time

decay series series of nuclear decays ending in a stable nucleus

fission splitting of a nucleus

gamma (γ) rays one of the types of rays emitted from the nucleus of an atom as gamma particles

gamma decay radioactive nuclear decay associated with the emission of gamma radiation

half-life time for half of the original nuclei to decay (or half of the original nuclei remain)

high dose dose of radiation greater than 1 Sv (100 rem)

isotopes nuclei having the same number of protons but different numbers of neutrons

lifetime average time that a nucleus exists before decaying

liquid drop model model of nucleus (only to understand some of its features) in which nucleons in a nucleus act like atoms in a drop

low dose dose of radiation less than 100 mSv (10 rem)

mass defect difference between the mass of a nucleus and the total mass of its constituent nucleons

mass number number of nucleons in a nucleus

moderate dose dose of radiation from 0.1 Sv to 1 Sv (10 to 100 rem)

neutrino subatomic elementary particle which has no net electric charge

neutron number number of neutrons in a nucleus

nuclear fusion process of combining lighter nuclei to make heavier nuclei

nuclear fusion reactor nuclear reactor that uses the fusion chain to produce energy

nucleons protons and neutrons found inside the nucleus of an atom

nucleosynthesis process of fusion by which all elements on Earth are believed to have been created

nuclide nucleus

parent nucleus original nucleus before decay

positron electron with positive charge

positron emission tomography (PET) tomography technique that uses β^+ emitters and detects the two annihilation γ rays, aiding in source localization

proton-proton chain combined reactions that fuse hydrogen nuclei to produce He nuclei

radiation dose unit (rad) ionizing energy deposited per kilogram of tissue

radioactive dating application of radioactive decay in which the age of a material is determined by the amount of radioactivity of a particular type that occurs

radioactive decay law describes the exponential decrease of parent nuclei in a radioactive sample

radioactive tags special drugs (radiopharmaceuticals) that allow doctors to track movement of other drugs in the body

radioactivity spontaneous emission of radiation from nuclei

radiopharmaceutical compound used for medical imaging

radius of a nucleus radius of a nucleus is defined as $r = r_0 A^{1/3}$

relative biological effectiveness (RBE) number that expresses the relative amount of damage that a fixed amount of ionizing radiation of a given type can inflict on biological tissues

roentgen equivalent man (rem) dose unit more closely related to effects in biological tissue

sievert (Sv) SI equivalent of the rem

single-photon-emission computed tomography (SPECT) tomography performed with γ-emitting radiopharmaceuticals

strong nuclear force force that binds nucleons together in the nucleus

transuranic element element that lies beyond uranium in the periodic table

KEY EQUATIONS

Atomic mass number	$A = Z + N$
Standard format for expressing an isotope	${}^{A}_{Z}\text{X}$
Nuclear radius, where r_0 is the radius of a single proton	$r = r_0 A^{1/3}$
Mass defect	$\Delta m = Zm_p + (A - Z)m_n - m_{\text{nuc}}$

Binding energy	$E = (\Delta m)c^2$
Binding energy per nucleon	$BEN = \frac{E_b}{A}$
Radioactive decay rate	$-\frac{dN}{dt} = \lambda N$
Radioactive decay law	$N = N_0 e^{-\lambda t}$
Decay constant	$\lambda = \frac{0.693}{T_{1/2}}$
Lifetime of a substance	$\bar{T} = \frac{1}{\lambda}$
Activity of a radioactive substance	$A = A_0 e^{-\lambda t}$
Activity of a radioactive substance (linear form)	$\ln A = -\lambda t + \ln A_0$
Alpha decay	${}^{A}_{Z}\text{X} \rightarrow {}^{A-4}_{Z-2}\text{X} + {}^{4}_{2}\text{He}$
Beta decay	${}^{A}_{Z}\text{X} \rightarrow {}_{Z+1}{}^{A}\text{X} + {}_{-1}^{0}\text{e} + \bar{\nu}$
Positron emission	${}^{A}_{Z}\text{X} \rightarrow {}_{Z-1}{}^{A}\text{X} + {}_{+1}^{0}\text{e} + \nu$
Gamma decay	${}^{A}_{Z}\text{X}^* \rightarrow {}^{A}_{Z}\text{X} + \gamma$

SUMMARY

10.1 Properties of Nuclei

- The atomic nucleus is composed of protons and neutrons.
- The number of protons in the nucleus is given by the atomic number, Z. The number of neutrons in the nucleus is the neutron number, N. The number of nucleons is mass number, A.
- Atomic nuclei with the same atomic number, Z, but different neutron numbers, N, are isotopes of the same element.
- The atomic mass of an element is the weighted average of the masses of its isotopes.

10.2 Nuclear Binding Energy

- The mass defect of a nucleus is the difference between the total mass of a nucleus and the sum of the masses of all its constituent nucleons.
- The binding energy (BE) of a nucleus is equal to the amount of energy released in forming the nucleus, or the mass defect multiplied by the speed of light squared.
- A graph of binding energy per nucleon (BEN) versus atomic number A implies that nuclei divided or combined release an enormous amount of energy.
- The binding energy of a nucleon in a nucleus is analogous to the ionization energy of an electron in an atom.

10.3 Radioactive Decay

- In the decay of a radioactive substance, if the decay constant (λ) is large, the half-life is small, and vice versa.
- The radioactive decay law, $N = N_0 e^{-\lambda t}$, uses the properties of radioactive substances to estimate the age of a substance.
- Radioactive carbon has the same chemistry as stable carbon, so it mixes into the ecosphere and eventually becomes

part of every living organism. By comparing the abundance of ^{14}C in an artifact with the normal abundance in living tissue, it is possible to determine the artifact's age.

10.4 Nuclear Reactions

- The three types of nuclear radiation are alpha (α) rays, beta (β) rays, and gamma (γ) rays.
- We represent α decay symbolically by ${}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}X + {}^{4}_{2}He$. There are two types of β decay: either an electron (β^-) or a positron (β^+) is emitted by a nucleus. γ decay is represented symbolically by ${}^{A}_{Z}X^* \rightarrow {}^{A}_{Z}X + \gamma$.
- When a heavy nucleus decays to a lighter one, the lighter daughter nucleus can become the parent nucleus for the next decay, and so on, producing a decay series.

10.5 Fission

- Nuclear fission is a process in which the sum of the masses of the product nuclei are less than the masses of the reactants.
- Energy changes in a nuclear fission reaction can be understood in terms of the binding energy per nucleon curve.
- The production of new or different isotopes by nuclear transformation is called breeding, and reactors designed for this purpose are called breeder reactors.

10.6 Nuclear Fusion

- Nuclear fusion is a reaction in which two nuclei are combined to form a larger nucleus; energy is released when light nuclei are fused to form medium-mass nuclei.
- The amount of energy released by a fusion reaction is known as the *Q* value.
- Nuclear fusion explains the reaction between deuterium and tritium that produces a fusion (or hydrogen) bomb; fusion also explains the production of energy in the Sun, the process of nucleosynthesis, and the creation of the heavy elements.

10.7 Medical Applications and Biological Effects of Nuclear Radiation

- Nuclear technology is used in medicine to locate and study diseased tissue using special drugs called radiopharmaceuticals. Radioactive tags are used to identify cancer cells in the bones, brain tumors, and Alzheimer's disease, and to monitor the function of body organs, such as blood flow, heart muscle activity, and iodine uptake in the thyroid gland.
- The biological effects of ionizing radiation are due to two effects it has on cells: interference with cell reproduction and destruction of cell function.
- Common sources of radiation include that emitted by Earth due to the isotopes of uranium, thorium, and potassium; natural radiation from cosmic rays, soils, and building materials, and artificial sources from medical and dental diagnostic tests.
- Biological effects of nuclear radiation are expressed by many different physical quantities and in many different units, including the rad or radiation dose unit.

CONCEPTUAL QUESTIONS

10.1 Properties of Nuclei

1. Define and make clear distinctions between the terms neutron, nucleon, nucleus, and nuclide.

2. What are isotopes? Why do isotopes of the same atom share the same chemical properties?

10.2 Nuclear Binding Energy

3. Explain why a bound system should have less mass than its components. Why is this not observed traditionally, say, for a building made of bricks?

4. Why is the number of neutrons greater than the number of protons in stable nuclei that have an *A* greater than about 40? Why is this effect more pronounced for the heaviest nuclei?

5. To obtain the most precise value of the binding energy per nucleon, it is important to take into account forces between nucleons at the surface of the nucleus. Will surface effects increase or decrease estimates of BEN?

10.3 Radioactive Decay

6. How is the initial activity rate of a radioactive substance related to its half-life?

7. For the carbon dating described in this chapter, what important assumption is made about the time variation in the intensity of cosmic rays?

10.4 Nuclear Reactions

8. What is the key difference and the key similarity between beta (β^-) decay and alpha decay?

9. What is the difference between γ rays and characteristic X-rays and visible light?

10. What characteristics of radioactivity show it to be nuclear in origin and not atomic?

11. Consider Figure 10.12. If the magnetic field is replaced by an electric field pointed in toward the page, in which directions will the α-, β^+ -, and γ rays bend?

12. Why is Earth's core molten?

10.5 Fission

13. Should an atomic bomb really be called *nuclear* bomb?

14. Why does a chain reaction occur during a fission reaction?

15. In what way is an atomic nucleus like a liquid drop?

10.6 Nuclear Fusion

16. Explain the difference between nuclear fission and nuclear fusion.

17. Why does the fusion of light nuclei into heavier nuclei release energy?

10.7 Medical Applications and Biological Effects of Nuclear Radiation

18. Why is a PET scan more accurate than a SPECT scan?

19. Isotopes that emit α radiation are relatively safe outside the body and exceptionally hazardous inside. Explain why.

20. Ionizing radiation can impair the ability of a cell to repair DNA. What are the three ways the cell can respond?

PROBLEMS

10.1 Properties of Nuclei

21. Find the atomic numbers, mass numbers, and neutron numbers for (a) ${}^{58}_{29}\text{Cu}$, (b) ${}^{24}_{11}\text{Na}$, (c) ${}^{210}_{84}\text{Po}$, (d) ${}^{45}_{20}\text{Ca}$, and (e) ${}^{206}_{82}\text{Pb}$.

22. Silver has two stable isotopes. The nucleus, ${}^{107}_{47}\text{Ag}$, has atomic mass 106.905095 g/mol with an abundance of 51.83% ; whereas ${}^{109}_{47}\text{Ag}$ has atomic mass 108.904754 g/mol with an abundance of 48.17% . Find the atomic mass of the element silver.

23. The mass (*M*) and the radius (*r*) of a nucleus can be expressed in terms of the mass number, *A*. (a) Show that the density of a nucleus is independent of *A*. (b) Calculate the density of a gold (Au) nucleus. Compare your answer to that for iron (Fe).

24. A particle has a mass equal to 10 u. If this mass is converted completely into energy, how much energy is released? Express your answer in mega-electron volts (MeV). (Recall that $1\text{ eV} = 1.6 \times 10^{-19}\text{ J}$.)

25. Find the length of a side of a cube having a mass of 1.0 kg and the density of nuclear matter.

26. The detail that you can observe using a probe is limited by its wavelength. Calculate the energy of a particle that has a wavelength of $1 \times 10^{-16}\text{ m}$, small enough to detect details about one-tenth the size of a nucleon.

10.2 Nuclear Binding Energy

27. How much energy would be released if six hydrogen atoms and six neutrons were combined to form $^{12}_{6}C$?

28. Find the mass defect and the binding energy for the helium-4 nucleus.

29. ^{56}Fe is among the most tightly bound of all nuclides. It makes up more than 90% of natural iron. Note that ^{56}Fe has even numbers of protons and neutrons. Calculate the binding energy per nucleon for ^{56}Fe and compare it with the approximate value obtained from the graph in **Figure 10.7**.

30. ^{209}Bi is the heaviest stable nuclide, and its BEN is low compared with medium-mass nuclides. Calculate BEN for this nucleus and compare it with the approximate value obtained from the graph in **Figure 10.7**.

31. (a) Calculate BEN for ^{235}U, the rarer of the two most common uranium isotopes; (b) Calculate BEN for ^{238}U. (Most of uranium is ^{238}U.)

32. The fact that BEN peaks at roughly $A = 60$ implies that the *range* of the strong nuclear force is about the diameter of this nucleus.

(a) Calculate the diameter of $A = 60$ nucleus.

(b) Compare BEN for ^{58}Ni and ^{90}Sr. The first is one of the most tightly bound nuclides, whereas the second is larger and less tightly bound.

10.3 Radioactive Decay

33. A sample of radioactive material is obtained from a very old rock. A plot ln*A* verses *t* yields a slope value of $-10^{-9}\ s^{-1}$ (see **Figure 10.10**(b)). What is the half-life of this material?

34. Show that: $\bar{T} = \frac{1}{\lambda}$.

35. The half-life of strontium-91, $^{91}_{38}Sr$ is 9.70 h. Find (a) its decay constant and (b) for an initial 1.00-g sample, the activity after 15 hours.

36. A sample of pure carbon-14 $(T_{1/2} = 5730\ y)$ has an activity of $1.0\ \mu Ci$. What is the mass of the sample?

37. A radioactive sample initially contains 2.40×10^{-2} mol of a radioactive material whose half-life is 6.00 h. How many moles of the radioactive material remain after 6.00 h? After 12.0 h? After 36.0 h?

38. An old campfire is uncovered during an archaeological dig. Its charcoal is found to contain less than 1/1000 the normal amount of ^{14}C. Estimate the minimum age of the charcoal, noting that $2^{10} = 1024$.

39. Calculate the activity R, in curies of 1.00 g of ^{226}Ra. (b) Explain why your answer is not exactly 1.00 Ci, given that the curie was originally supposed to be exactly the activity of a gram of radium.

40. Natural uranium consists of ^{235}U (percent abundance $= 0.7200\%$, $\lambda = 3.12 \times 10^{-17}/s$) and ^{238}U (percent abundance $= 99.27\%$, $\lambda = 4.92 \times 10^{-18}/s$). What were the values for percent abundance of ^{235}U and ^{238}U when Earth formed 4.5×10^9 years ago?

41. World War II aircraft had instruments with glowing radium-painted dials. The activity of one such instrument was 1.0×10^5 Bq when new. (a) What mass of ^{226}Ra was present? (b) After some years, the phosphors on the dials deteriorated chemically, but the radium did not escape. What is the activity of this instrument 57.0 years after it was made?

42. The ^{210}Po source used in a physics laboratory is labeled as having an activity of $1.0\ \mu Ci$ on the date it was prepared. A student measures the radioactivity of this source with a Geiger counter and observes 1500 counts per minute. She notices that the source was prepared 120 days before her lab. What fraction of the decays is she observing with her apparatus?

43. Armor-piercing shells with depleted uranium cores are fired by aircraft at tanks. (The high density of the uranium makes them effective.) The uranium is called depleted because it has had its ^{235}U removed for reactor use and is nearly pure ^{238}U. Depleted uranium has been erroneously called nonradioactive. To demonstrate that this is wrong: (a) Calculate the activity of 60.0 g of pure ^{238}U. (b) Calculate the activity of 60.0 g of natural uranium, neglecting the ^{234}U and all daughter nuclides.

10.4 Nuclear Reactions

44. ^{249}Cf undergoes alpha decay. (a) Write the reaction equation. (b) Find the energy released in the decay.

45. (a) Calculate the energy released in the α decay of ^{238}U. (b) What fraction of the mass of a single ^{238}U is destroyed in the decay? The mass of ^{234}Th is 234.043593 u. (c) Although the fractional mass loss is large for a single nucleus, it is difficult to observe for an entire macroscopic sample of uranium. Why is this?

46. The β^- particles emitted in the decay of ^3H (tritium) interact with matter to create light in a glow-in-the-dark exit sign. At the time of manufacture, such a sign contains 15.0 Ci of ^3H. (a) What is the mass of the tritium? (b) What is its activity 5.00 y after manufacture?

47. (a) Write the complete β^- decay equation for ^{90}Sr, a major waste product of nuclear reactors. (b) Find the energy released in the decay.

48. Write a nuclear β^- decay reaction that produces the ^{90}Y nucleus. (*Hint:* The parent nuclide is a major waste product of reactors and has chemistry similar to calcium, so that it is concentrated in bones if ingested.)

49. Write the complete decay equation in the complete $^A_Z\text{X}_N$ notation for the beta (β^-) decay of ^3H (tritium), a manufactured isotope of hydrogen used in some digital watch displays, and manufactured primarily for use in hydrogen bombs.

50. If a 1.50-cm-thick piece of lead can absorb 90.0% of the rays from a radioactive source, how many centimeters of lead are needed to absorb all but 0.100% of the rays?

51. An electron can interact with a nucleus through the beta-decay process:

$$^A_Z\text{X} + e^- \rightarrow Y + v_e .$$

(a) Write the complete reaction equation for electron capture by ^7Be.

(b) Calculate the energy released.

52. (a) Write the complete reaction equation for electron capture by ^{15}O.

(b) Calculate the energy released.

53. A rare decay mode has been observed in which ^{222}Ra emits a ^{14}C nucleus. (a) The decay equation is $^{222}\text{Ra} \rightarrow {}^A\text{X} + {}^{14}\text{C}$. Identify the nuclide ^AX. (b) Find the energy emitted in the decay. The mass of ^{222}Ra is 222.015353 u.

10.5 Fission

54. A large power reactor that has been in operation for some months is turned off, but residual activity in the core still produces 150 MW of power. If the average energy per decay of the fission products is 1.00 MeV, what is the core activity?

55. (a) Calculate the energy released in the neutron-induced fission $n + {}^{238}\text{U} \rightarrow {}^{96}\text{Sr} + {}^{140}\text{Xe} + 3n$, given $m\left({}^{96}\text{Sr}\right) = 95.921750\text{ u}$ and $m\left({}^{140}\text{Xe}\right) = 139.92164$.

(b) This result is about 6 MeV greater than the result for spontaneous fission. Why?

(c) Confirm that the total number of nucleons and total charge are conserved in this reaction.

56. (a) Calculate the energy released in the neutron-induced fission reaction $n + {}^{235}\text{U} \rightarrow {}^{92}\text{Kr} + {}^{142}\text{Ba} + 2n$, given $m\left({}^{92}\text{Kr}\right) = 91.926269\text{ u}$ and $m\left({}^{142}\text{Ba}\right) = 141.916361\text{ u}$. (b) Confirm that the total number of nucleons and total charge are conserved in this reaction.

57. The electrical power output of a large nuclear reactor facility is 900 MW. It has a 35.0% efficiency in converting nuclear power to electrical power.

(a) What is the thermal nuclear power output in megawatts?

(b) How many ^{235}U nuclei fission each second, assuming the average fission produces 200 MeV?

(c) What mass of ^{235}U is fissioned in 1 year of full-power operation?

58. Find the total energy released if 1.00 kg of $^{235}_{92}\text{U}$ were to undergo fission.

10.6 Nuclear Fusion

59. Verify that the total number of nucleons, and total charge are conserved for each of the following fusion reactions in the proton-proton chain.

(i) $^{1}\text{H} + {}^{1}\text{H} \rightarrow {}^{2}\text{H} + e^{+} + v_{\text{e}}$,

(ii) $^{1}\text{H} + {}^{2}\text{H} \rightarrow {}^{3}\text{He} + \gamma$, and (iii) $^{3}\text{He} + {}^{3}\text{He} \rightarrow {}^{4}\text{He} + {}^{1}\text{H} + {}^{1}\text{H}$.

(List the value of each of the conserved quantities before and after each of the reactions.)

60. Calculate the energy output in each of the fusion reactions in the proton-proton chain, and verify the values determined in the preceding problem.

61. Show that the total energy released in the proton-proton chain is 26.7 MeV, considering the overall effect in $^{1}\text{H} + {}^{1}\text{H} \rightarrow {}^{2}\text{H} + e^{+} + v_{\text{e}}$, $^{1}\text{H} + {}^{2}\text{H} \rightarrow {}^{3}\text{He} + \gamma$, and $^{3}\text{He} + {}^{3}\text{He} \rightarrow {}^{4}\text{He} + {}^{1}\text{H} + {}^{1}\text{H}$. Be sure to include the annihilation energy.

62. Two fusion reactions mentioned in the text are $n + {}^{3}\text{He} \rightarrow {}^{4}\text{He} + \gamma$ and $n + {}^{1}\text{H} \rightarrow {}^{2}\text{H} + \gamma$. Both reactions release energy, but the second also creates more fuel. Confirm that the energies produced in the reactions are 20.58 and 2.22 MeV, respectively. Comment on which product nuclide is most tightly bound, ^{4}He or ^{2}H.

63. The power output of the Sun is $4 \times 10^{26}\,\text{W}$. (a) If 90% of this energy is supplied by the proton-proton chain, how many protons are consumed per second? (b) How many neutrinos per second should there be per square meter at the surface of Earth from this process?

64. Another set of reactions that fuses hydrogen into helium in the Sun and especially in hotter stars is called the CNO cycle:

$$^{12}\text{C} + {}^{1}\text{H} \rightarrow {}^{13}\text{N} + \gamma$$

$$^{13}\text{N} \rightarrow {}^{13}\text{C} + e^{+} + v_{\text{e}}$$

$$^{13}\text{C} + {}^{1}\text{H} \rightarrow {}^{14}\text{N} + \gamma$$

$$^{14}\text{N} + {}^{1}\text{H} \rightarrow {}^{15}\text{O} + \gamma$$

$$^{15}\text{O} \rightarrow {}^{15}\text{N} + e^{+} + v_{\text{e}}$$

$$^{15}\text{N} + {}^{1}\text{H} \rightarrow {}^{12}\text{C} + {}^{4}\text{He}$$

This process is a “cycle” because ^{12}C appears at the beginning and end of these reactions. Write down the overall effect of this cycle (as done for the proton-proton chain in $2e^{-} + 4{}^{1}\text{H} \rightarrow {}^{4}\text{He} + 2v_{\text{e}} + 6\gamma$). Assume that the positrons annihilate electrons to form more γ rays.

65. (a) Calculate the energy released by the fusion of a 1.00-kg mixture of deuterium and tritium, which produces helium. There are equal numbers of deuterium and tritium nuclei in the mixture.

(b) If this process takes place continuously over a period of a year, what is the average power output?

10.7 Medical Applications and Biological Effects of Nuclear Radiation

66. What is the dose in mSv for: (a) a 0.1-Gy X-ray? (b) 2.5 mGy of neutron exposure to the eye? (c) 1.5m Gy of α exposure?

67. Find the radiation dose in Gy for: (a) A 10-mSv fluoroscopic X-ray series. (b) 50 mSv of skin exposure by an α emitter. (c) 160 mSv of β^{-} and γ rays from the ^{40}K in your body.

68. Find the mass of ^{239}Pu that has an activity of $1.00\,\mu\text{Ci}$.

69. In the 1980s, the term picowave was used to describe food irradiation in order to overcome public resistance by playing on the well-known safety of microwave radiation. Find the energy in MeV of a photon having a wavelength of a picometer.

70. What is the dose in Sv in a cancer treatment that exposes the patient to 200 Gy of γ rays?

71. One half the γ rays from $^{99\text{m}}\text{Tc}$ are absorbed by a 0.170-mm-thick lead shielding. Half of the γ rays that pass through the first layer of lead are absorbed in a second layer of equal thickness. What thickness of lead will absorb all but one in 1000 of these γ rays?

72. How many Gy of exposure is needed to give a cancerous tumor a dose of 40 Sv if it is exposed to α activity?

73. A plumber at a nuclear power plant receives a whole-body dose of 30 mSv in 15 minutes while repairing a crucial valve. Find the radiation-induced yearly risk of death from cancer and the chance of genetic defect from this maximum allowable exposure.

74. Calculate the dose in rem/y for the lungs of a weapons plant employee who inhales and retains an activity of $1.00\mu\text{Ci}$ ^{239}Pu in an accident. The mass of affected lung tissue is 2.00 kg and the plutonium decays by emission of a

5.23-MeV α particle. Assume a RBE value of 20.

ADDITIONAL PROBLEMS

75. The wiki-phony site states that the atomic mass of chlorine is 40 g/mol. Check this result. *Hint:* The two, most common stable isotopes of chlorine are: $^{35}_{17}\text{Cl}$ and $^{37}_{17}\text{Cl}$. (The abundance of Cl-35 is 75.8%, and the abundance of Cl-37 is 24.2%.)

76. A particle physicist discovers a neutral particle with a mass of 2.02733 u that he assumes is two neutrons bound together.

(a) Find the binding energy.

(b) What is unreasonable about this result?

77. A nuclear physicist finds $1.0\,\mu g$ of ^{236}U in a piece of uranium ore ($T_{1/2} = 2.348 \times 10^7\,\text{y}$). (a) Use the decay law to determine how much ^{236}U would had to have been on Earth when it formed $4.543 \times 10^9\,\text{y}$ ago for $1.0\,\mu g$ to be left today. (b) What is unreasonable about this result? (c) How is this unreasonable result resolved?

78. A group of scientists use carbon dating to date a piece of wood to be 3 billion years old. Why doesn't this make sense?

79. According to your lab partner, a 2.00-cm-thick sodium-iodide crystal absorbs all but 10% of rays from a radioactive source and a 4.00-cm piece of the same material absorbs all but 5%? Is this result reasonable?

80. In the science section of the newspaper, an article reports the efforts of a group of scientists to create a new nuclear reactor based on the fission of iron (Fe). Is this a good idea?

81. The ceramic glaze on a red-orange "Fiestaware" plate is U_2O_3 and contains 50.0 grams of ^{238}U, but very little ^{235}U. (a) What is the activity of the plate? (b) Calculate the total energy that will be released by the ^{238}U decay. (c) If energy is worth 12.0 cents per $\text{kW} \cdot \text{h}$, what is the monetary value of the energy emitted? (These brightly-colored ceramic plates went out of production some 30 years ago, but are still available as collectibles.)

82. Large amounts of depleted uranium $\left(^{238}\text{U}\right)$ are available as a by-product of uranium processing for reactor fuel and weapons. Uranium is very dense and makes good counter weights for aircraft. Suppose you have a 4000-kg block of ^{238}U. (a) Find its activity. (b) How many calories per day are generated by thermalization of the decay energy? (c) Do you think you could detect this as heat? Explain.

83. A piece of wood from an ancient Egyptian tomb is tested for its carbon-14 activity. It is found to have an activity per gram of carbon of $A = 10\,\text{decay/min} \cdot \text{g}$. What is the age of the wood?

CHALLENGE PROBLEMS

84. This problem demonstrates that the binding energy of the electron in the ground state of a hydrogen atom is much smaller than the rest mass energies of the proton and electron.

(a) Calculate the mass equivalent in u of the 13.6-eV binding energy of an electron in a hydrogen atom, and compare this with the known mass of the hydrogen atom.

(b) Subtract the known mass of the proton from the known mass of the hydrogen atom.

(c) Take the ratio of the binding energy of the electron (13.6 eV) to the energy equivalent of the electron's mass (0.511 MeV).

(d) Discuss how your answers confirm the stated purpose of this problem.

85. The *Galileo* space probe was launched on its long journey past Venus and Earth in 1989, with an ultimate goal of Jupiter. Its power source is 11.0 kg of ^{238}Pu, a by-product of nuclear weapons plutonium production. Electrical energy is generated thermoelectrically from the heat produced when the 5.59-MeV α particles emitted in each decay crash to a halt inside the plutonium and its shielding. The half-life of ^{238}Pu is 87.7 years.

(a) What was the original activity of the ^{238}Pu in becquerels?

(b) What power was emitted in kilowatts?

(c) What power was emitted 12.0 y after launch? You may neglect any extra energy from daughter nuclides and any losses from escaping γ rays.

86. Find the energy emitted in the β^- decay of ^{60}Co.

87. Engineers are frequently called on to inspect and, if necessary, repair equipment in nuclear power plants. Suppose that the city lights go out. After inspecting the nuclear reactor, you find a leaky pipe that leads from the steam generator to turbine chamber. (a) How do the pressure readings for the turbine chamber and steam condenser compare? (b) Why is the nuclear reactor *not* generating electricity?

88. If two nuclei are to fuse in a nuclear reaction, they must be moving fast enough so that the repulsive Coulomb force between them does not prevent them for getting within $R \approx 10^{-14}\,\text{m}$ of one another. At this distance or nearer, the attractive nuclear force can overcome the Coulomb force, and the nuclei are able to fuse.

(a) Find a simple formula that can be used to estimate the minimum kinetic energy the nuclei must have if they are to fuse. To keep the calculation simple, assume the two nuclei are identical and moving toward one another with the same speed *v*. (b) Use this minimum kinetic energy to estimate the minimum temperature a gas of the nuclei must have before a significant number of them will undergo fusion. Calculate this minimum temperature first for hydrogen and then for helium. (*Hint:* For fusion to occur, the minimum kinetic energy when the nuclei are far apart must be equal to the Coulomb potential energy when they are a distance *R* apart.)

89. For the reaction, $n + {}^3\text{He} \rightarrow {}^4\text{He} + \gamma$, find the amount of energy transfers to ^4He and γ (on the right side of the equation). Assume the reactants are initially at rest. (*Hint:* Use conservation of momentum principle.)

90. Engineers are frequently called on to inspect and, if necessary, repair equipment in medical hospitals. Suppose that the PET system malfunctions. After inspecting the unit, you suspect that one of the PET photon detectors is misaligned. To test your theory you position one detector at the location $(r, \theta, \varphi) = (1.5, 45, 30)$ relative to a radioactive test sample at the center of the patient bed. (a) If the second photon detector is properly aligned where should it be located? (b) What energy reading is expected?