Linear Momentum and Collisions: Conservation of Momentum and Collisions

Learning Objectives

By the end of this section, students will be able to:

- Explain the meaning of “conservation of momentum”
- Correctly identify if a system is, or is not, closed
- Define a system whose momentum is conserved
- Mathematically express the conservation of momentum for a given system
- Calculate an unknown quantity using conservation of momentum
- Identify the type of collision
- Correctly label a collision as elastic or inelastic
- Use kinetic energy along with momentum and impulse to analyze a collision

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Conservation of Momentum & Collisions

Types of Collisions & Elastic Collisions

Conservation of momentum is an important tool while studying interactions between individual objects in an **isolated system**. In particular, it is a useful tool to study collisions between different objects within a system.

In this section, we consider collisions of particles **inside a system when no external forces (negligible external forces)** act on the system and the mass of the system is constant. Momentum is **conserved** in these types of collisions.
Collisions are classified into two categories based on whether the internal energy (sum of individual kinetic energies of all objects in the system) is also conserved in addition to linear momentum.

Elastic Collision
Collision where $\vec{P}_{system}$ is conserved, and internal energy is also conserved.

Inelastic Collision
Collision where $\vec{P}_{system}$ is conserved, but internal energy is not conserved.

Completely Inelastic Collision
This is a special case of an inelastic collision, where colliding objects stick to one another after the collision and move as a single object

1. Elastic Collision

In an elastic collision:

- $\vec{P}_{system}$ is conserved.
- Total internal energy is conserved.

Truly elastic collisions can only be achieved with subatomic particles, such as electrons striking nuclei. Macroscopic collisions can be very nearly, but not quite, elastic—some kinetic energy is always converted into other forms of energy, such as heat transfer due to friction and sound. One macroscopic collision that is nearly elastic is that of two steel blocks on ice. Another nearly elastic collision is that between two carts with spring bumpers on an air track. Icy surfaces and air tracks are nearly frictionless, more readily allowing nearly elastic collisions on them.

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Consider an elastic collision between two masses m_1 and m_2. Their velocities before and after collisions are shown in Figure 1.

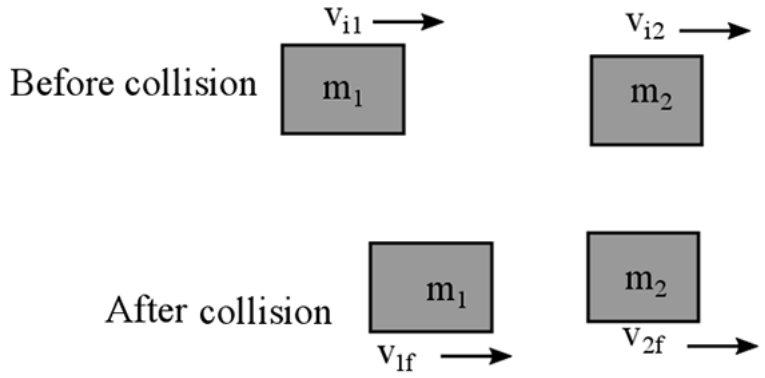


Figure 1: Two masses colliding

From conservation of momentum, we can write.

$$m_1 v_{i1} + m_2 v_{i2} = m_1 v_{f1} + m_2 v_{f2} \quad (1)$$

From conservation of internal energy, total KE = constant. WE can express this condition as follows.

$$\frac{1}{2} m_1 v_{i1}^2 + \frac{1}{2} m_2 v_{i2}^2 = \frac{1}{2} m_1 v_{f_1}^2 + \frac{1}{2} m_2 v_{f2}^2 \quad (2)$$

Canceling ½ and rearranging (2), we have

$$m_1\left(v_{i1}^2 - v_{f1}^2\right) = m_2\left(v_{f2}^2 - v_{i1}^2\right)$$

Using $a^2 - b^2 = (a+b)(a-b)$

$$m_1\left(v_{i1} + v_{f1}\right)\left(v_{i1} - v_{f1}\right) = m_2\left(v_{f2} + v_{i2}\right)\left(v_{f2} - v_{i2}\right) \quad (3)$$

Recall (1)

$$m_1 v_{i1} + m_2 v_{i2} = m_1 v_{f1} + m_2 v_{f2} \quad (1)$$

Rearrange (1) to collect terms of object 1 on the left side and object 2 on the right side.

$$m_1\left(v_{i1} - v_{f1}\right) = m_2\left(v_{f2} - v_{i2}\right). \quad (4)$$

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Substituting (4) in (3)

$$\left(v_{f1} + v_{i1}\right) = \left(v_{f2} + v_{i2}\right) \quad (5)$$

Rearranging (5), we get

$$v_{i1} - v_{i2} = v_{f2} - v_{f1} = -\left(v_{f1} - v_{f2}\right).$$

Identifying that $v_{i1} - v_{i2}$ equals the relative velocity of object 1 with respect to object 2 before collision, and $v_{f1} - v_{f2}$ equals the relative velocity of object 1 with respect to object 2 after collision, we conclude that in an elastic collision:

Relative velocity of mass 1 to 2 before collision = - Relative velocity of mass 1 with respect to 2 after collision

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While solving problems involving elastic collisions between two objects in one dimension, use one of the two sets of equations.

1. Linear Momentum is conserved:
$$m_1 v_{i1} + m_2 v_{i2} = m_1 v_{f1} + m_2 v_{f2} \quad (1)$$
2. Total internal energy (sum of kinetic energies) is constant.
$$\frac{1}{2} m_1 v_{i1}^2 + \frac{1}{2} m_2 v_{i2}^2 = \frac{1}{2} m_1 v_{f1}^2 + \frac{1}{2} m_2 v_{f2}^2 \quad (2)$$

Or,

1. Linear Momentum is conserved:
$$m_1 v_{i1} + m_2 v_{i2} = m_1 v_{f1} + m_2 v_{f2} \quad (1)$$
2. The relative velocities before collision are equal and opposite to the relative velocity of the objects after the collision.
$$v_{i1} - v_{i2} = -\left(v_{f1} - v_{f2}\right) \quad (2)$$

Self-Assess

What are the similarities and differences between elastic and inelastic collisions?

Answer: Linear momentum is conserved in both elastic and inelastic collisions. In addition, in elastic collisions, the total internal energy (total kinetic energy) is conserved, but not in inelastic collisions.

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1.2 Worked Example: Elastic Collisions

Calculate the velocities of two objects following an elastic collision, given that,

$$m_1 = 0.500\ kg, \quad m_2 = 3.50\ kg, \quad v_1 = 4.00\frac{m}{s}, \quad and\ v_2 = 0.$$

Strategy and Concept

First, visualize what the initial conditions mean—a small object strikes a larger object that is initially at rest. This situation is slightly simpler than the situation where both objects are initially moving. We are asked to find two unknowns (the final velocities v'_1 and v'_2). To find two unknowns, we must use two independent equations. Because this collision is elastic, we can use the above two equations. Both can be simplified by the fact that object 2 is initially at rest, and thus v_2=0. Once we simplify these equations, we combine them algebraically to solve for the unknowns.

Solution

For this problem, note that $v_2 = 0$ and use conservation of momentum. Thus,

$$p_1 = p'_1 + p'_2$$

Or

$$m_1 v_1 = m_1 v'_1 + m_2 v'_2.$$

Using conservation of internal kinetic energy and that $v_2 = 0$,

$$\frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_1 {v'_1}^2 + \frac{1}{2} m_2 {v'_2}^2.$$

Solving the first equation (momentum equation) for v'_2, we obtain

$$v'_2 = \frac{m_1}{m_2}(v_1 - v'_1).$$

Substituting this expression into the second equation (internal kinetic energy equation) eliminates the variable $v'2$, leaving only $v'1$ as an unknown (the algebra is left as an exercise for the reader). There are two solutions to any quadratic equation; in this example, they are

$$v'_1 = 4.00\frac{m}{s}$$

And

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$$v_1' = -3.0\frac{m}{s}$$

As noted when quadratic equations were encountered in earlier chapters, both solutions may or may not be meaningful. In this case, the first solution is the same as the initial condition. The first solution thus represents the situation before the collision and is discarded. The second solution (*v*'1=−3.00 m/s) is negative, meaning that the first object bounces backward. When this negative value of *v*'1 is used to find the velocity of the second object after the collision, we get

$$v_2' = \frac{m_1}{m_2}(v_1 - v_1') = \frac{0.500\ kg}{3.50\ kg}[4.00 - (-3.00)]\ m/s$$

Or

$$v_2' = 1.00\frac{m}{s}$$

Discussion

The result of this example is intuitively reasonable. A small object strikes a larger one at rest and bounces backward. The larger one is knocked forward, but at a low speed. (This is like a compact car bouncing backward off a full-size SUV that is initially at rest.) As a check, try calculating the internal kinetic energy before and after the collision. You will see that the internal kinetic energy is unchanged at 4.00 J. Also, check the total momentum before and after the collision; you will find it, too, is unchanged.

The equations for conservation of momentum and internal kinetic energy, as written above, can be used to describe any one-dimensional elastic collision of two objects. These equations can be extended to more objects if needed.

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2. Completely Inelastic Collision

Consider a system of two masses. The velocities of m_1 and m_2 before the collision are v_1 and v_2, respectively, as shown in Figure 2. After the collision, the two objects stick together and move with a common velocity v_f.

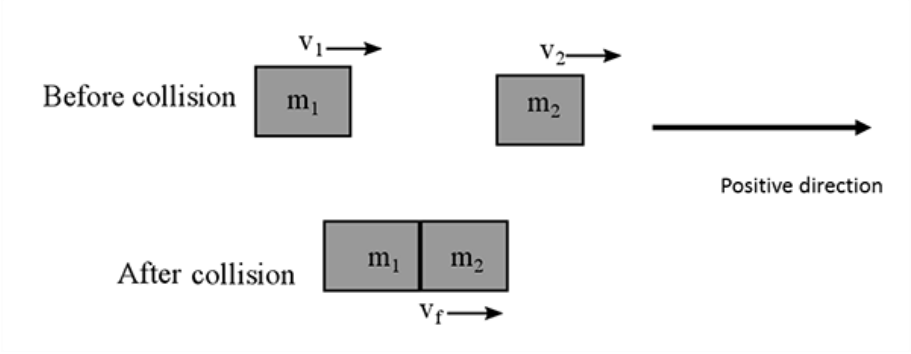


Figure 2: Completely inelastic collision

$$Initial\ Momentum = p_i = m_1 v_1 + m_2 v_2$$

$$Final\ Moment = p_f = (m_1 + m_2) v_f$$

If no external forces act on the system of the two masses, total momentum is conserved.

Note: In real life, friction is always present. But if the magnitude of the collisional forces is much greater than the frictional forces, we neglect friction and assume that external forces are not present. We can then apply the conservation of momentum. Figure 2 shows that the positive direction is to the right.

Applying the Law of Conservation of Momentum to the collision in Figure 2.

$$p_i = p_f$$

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v_f$$

Solving for v_f,

$$v_f = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

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Self-Assess

Define a completely inelastic collision.

Answer: A completely inelastic collision is a special case of inelastic collisions in which the objects combine to form a single object after the collision. Momentum of the system is conserved in a completely inelastic collision, but not the total kinetic energy.

2.1 Worked Example: Inelastic Collisions

Calculating Velocity and Change in Kinetic Energy: Inelastic Collision of a Puck and a Goalie

(a) Find the recoil velocity of a 70.0-kg ice hockey goalie, originally at rest, who catches a 0.150-kg hockey puck slapped at him at a velocity of 35.0 m/s.

(b) How much kinetic energy is lost during the collision? Assume friction between the ice and the puck-goalie system is negligible. (See Figure 3)

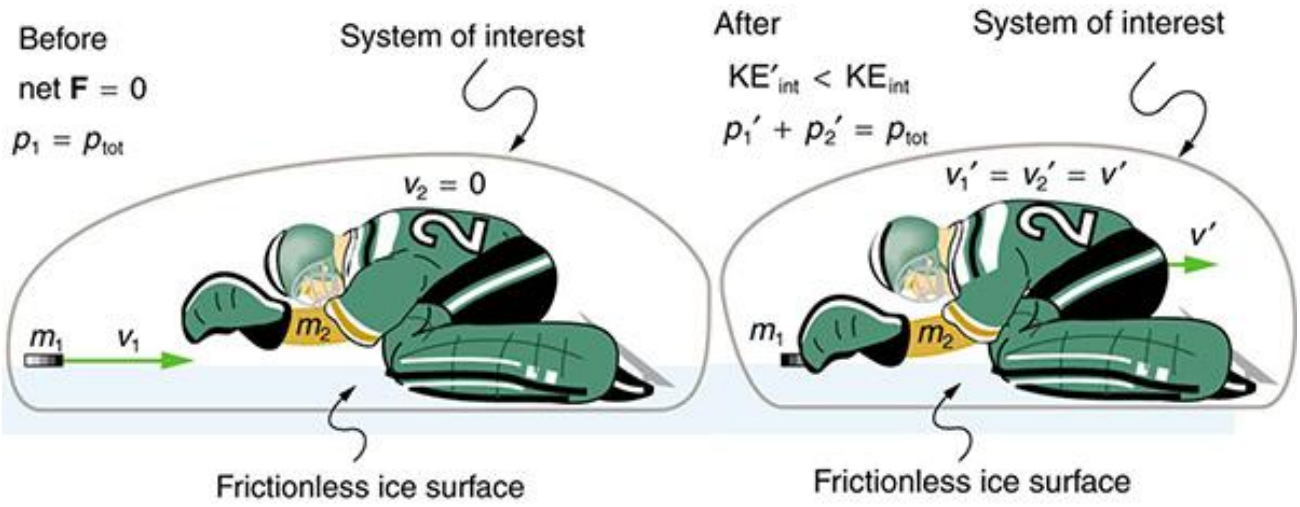


Figure 3: An ice hockey goalie catches a hockey puck and recoils backward. The initial kinetic energy of the puck is almost entirely converted to thermal energy and sound in this inelastic collision.

Strategy

Momentum is conserved because the net external force on the puck-goalie system is zero. We can thus use conservation of momentum to find the final velocity of the puck and goalie system. Note that the initial velocity of the goalie is zero and that the final velocity of the puck and the

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goalie are the same. Once the final velocity is found, the kinetic energies can be calculated before and after the collision and compared as requested.

Solution for (a)

Momentum is conserved because the net internal force on the puck-goalie system is zero.

Conservation of momentum is

$$p_1 + p_2 = p_1' + p_2'$$

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'.$$

Because the goalie is initially at rest, we know $v_2 = 0$. Because the goalie catches the puck, the final velocities are equal, or $v_1' = v_2' = v'$. Thus, the conservation of momentum equation simplifies to

$$m_1 v_1 = (m_1 m_2) v'$$

Solving for v' yields

$$v' = \frac{m_1}{m_1 + m_2} v_1$$

Entering known values in this equation, we get

$$v' = \frac{0.150\ kg}{0.150\ kg + 70.0\ kg}\left(35.0 \frac{m}{s}\right) = 7.48 \times 10^{-2} \frac{m}{s}.$$

Discussion for (a)

This recoil velocity is small and in the same direction as the puck's original velocity, as we might expect.

Solution for (b)

Before the collision, the internal kinetic energy KE_{int} of the system is that of the hockey puck, because the goalie is initially at rest. Therefore, KE_{int} is initially:

$$KE_{int} = \frac{1}{2} m v^2 = \frac{1}{2}(0.150\ kg)\left(35.0 \frac{m}{s}\right)^2$$

$$= 91.9\ J.$$

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3. Problem-Solving Strategies for Conservation of Momentum and Collision

1. Set-up and diagram:
    I. Choose a coordinate system
    II. Identify the quantities with appropriate signs for velocities
    III. Fix a notation. One recommendation for the notation is given below.
        a) Use a subscript to identify the object. For example, $_1$ for object 1, and so on.
        b) Use subscript "$_i$" for before collision/impact and "$_f$" for after collision/impact quantities.

        For example, v_{2i} or v_{i2} represent the initial velocity of the second object.
    IV. Draw two diagrams, one for system momenta before the collision/impact, and one for system momenta after the collision/impact.
2. Examine if any external forces are present. Decide if momentum is conserved in the collision or not.
3. If momentum is conserved, examine if the collision is elastic or inelastic
4. Apply the appropriate formulae to solve for the missing quantities.
5. Check the result for reasonableness with numbers.

Self-Assess

While solving problems involving collisions, it is helpful to use two subscripts to denote the velocity of the objects. What do the two subscripts represent?

Answer: It is helpful to use one subscript to denote the object and the other to denote the time, i.e, before the collision and after the collision.

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