Experiment

VIII Standing Waves on a String

I. Preparing for Lab

The purpose of this lab is to find the resonant frequency of the fundamental mode and its harmonics for a standing wave on a taut string, and to determine the propagation speed of the waves by analyzing the relationship of the wavelength and frequency of the waves.

To prepare for this lab before your session starts, read through the Physical Theory section below; for further reference, see sections 16.10 in your textbook.

*Finally, you must complete the **Pre-Lab questions** on **Expert TA** before your lab starts.*

Equipment:

string	tape measure	weight set
motor controller	precision balance	weight hangar
pulley-clamp assembly	variable-speed motor with eccentric shaft	
spreadsheet template for Experiment 8		

II. Physical Theory

When a transverse wave travels on a string this causes a displacement of the string in the transverse direction that varies with both time and position on the string. For a sinusoidal transverse wave travelling in the positive x-direction, the transverse displacement y of the string at time t and position x along the string is

$$y_1(x,t) = A\sin\left(\frac{2\pi}{\lambda}(x - vt)\right) = A\sin(kx - \omega t) \;, \tag{1}$$

where A is the **wave amplitude**, λ is the **wavelength,** v is the **wave propagation speed,** $k = 2\pi/\lambda$ is the **wave number**, and $\omega = 2\pi f$ is the **angular frequency**. From these expressions, one can show that v is related to the wavelength and frequency by

$$v = \lambda f \tag{2}$$

Wave speed is governed by properties of the string itself, and so equation (2) mutually constrains the values of λ and f against each other, i.e. if one is adjusted, the other will naturally change to preserve the product. Nonetheless, the propagation speed of a wave can be calculated by measuring both the frequency and wavelength of the same wave.

Consider a second wave on the same string, identical to the first in wavelength, frequency, and amplitude but traveling in the opposite direction:

$$y_2 = A\sin(kx + \omega t) \tag{3}$$

The resulting wave will be precisely the sum of the two waves at each point according to the **principal of superposition** (provided the amplitudes of the waves are not too large):

$$y_1 + y_2 = A\sin(kx - \omega t) + A\sin(kx + \omega t) = 2A\sin(kx)\cos(\omega t) \tag{4}$$

Note that without the subtraction of the space and time parts of the argument, the two terms kx and ωt do not act as mutual phase shifts on one another, and *the wave no longer seems to travel*; instead each point on the string undergoes simple oscillation according to $\cos(\omega t)$, moving up and down in unison (in phase), but with a **location-dependent amplitude** given by $2A\sin(kx)$. This kind of wave-like motion is aptly called a **standing wave**.

At points where kx is an odd multiple of $\pi/2$, $\sin(kx) = 1$, and the string has the maximum amplitude $2A$; these points are called "**anti-nodes**", for reasons which will become clear presently. At points where kx is a multiple of π, $\sin(kx) = 0$, and the string is perpetually stationary. These points are called "**nodes**". These properties can be summarized as follows:

Nodes ($y = 0$ at all times) $\quad kx = \dfrac{2\pi}{\lambda}x = n\pi; \quad n = 0, 1, 2, 3, \ldots$ (5)

Antinodes ($y_{max} = 2A$) $\quad kx = \dfrac{2\pi}{\lambda}x = 2(n+1)\pi; \quad n = 1, 2, 3, \ldots$ (6)

From equations (5) and (6) you can show that the distance between two adjacent nodes or two adjacent antinodes is $\lambda/2$; therefore one can find the wavelength by measuring the distance between adjacent nodes and multiplying by 2.

For a string that is fixed at both ends, in order for a standing wave to occur, it must be the case that the wave "fits" on the string such that there is a **node at each end**. Let the ends of the string be at $x = 0$ and $x = L$; then equation (5) indicates that the following condition must be met:

$$kx = \frac{2\pi L}{\lambda} = n\pi; \quad n = 0, 1, 2, 3, \ldots \tag{7}$$

Rearranging and simplify, we that there is a finite number of "allowed" wavelengths λ_n given by

$$\lambda_n = \frac{2L}{n}; \quad n = 1, 2, 3, \ldots \tag{8}$$

Figure 1 shows the shapes of the first four modes (with the four longest wavelengths).

Combining equation (8) with equation (2), we see that the resonant frequencies f_n of standing waves on a string of length L with wave speed v are

$$f_n = \frac{nv}{2L}; \quad n = 1, 2, 3, \ldots \tag{9}$$

The lowest resonant frequency is $f_n = v/2L$, corresponding to the oscillation mode with $n = 1$; this the **fundamental mode** or **1st harmonic**. According to equation (9) the frequencies of all higher harmonics are integer multiples of the fundamental frequency.

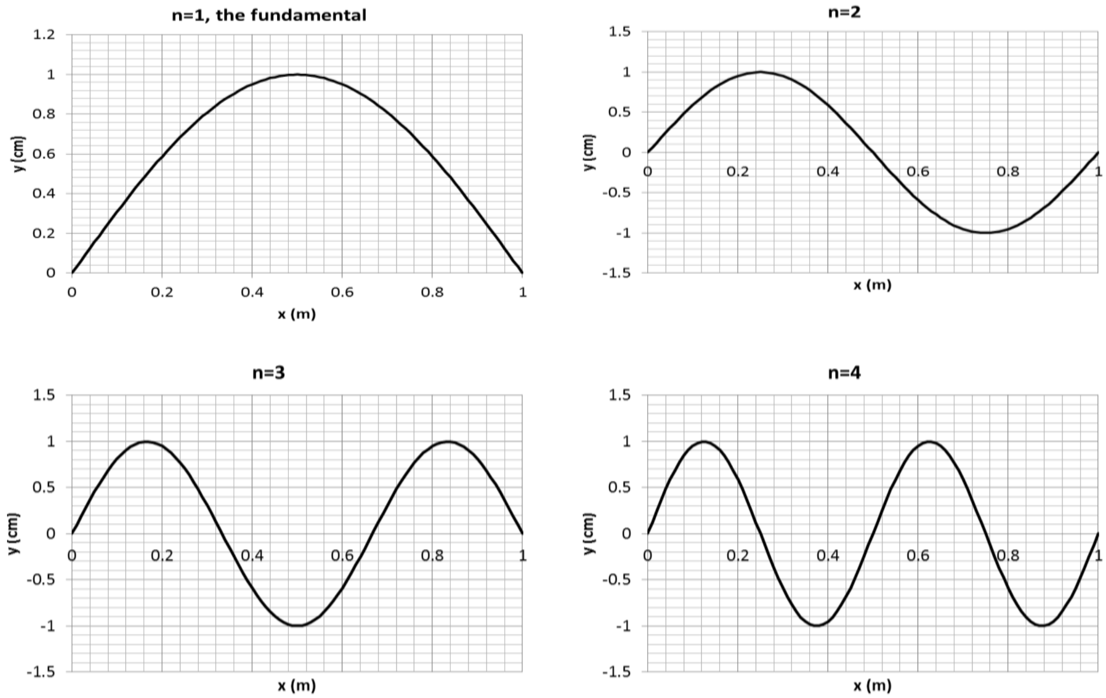

Figure 1. Plot-simulated, fixed-time snapshots showing displacement *y* for the first four modes or "harmonics of a standing wave on a string, as a function of position *x*.

For small amplitude waves on a string, the **wave speed** v is a function of the tension T in the string and its mass per unit length μ:

$$v = \sqrt{\frac{T}{\mu}} \tag{10}$$

From this, we can combine equations (9) and (10) to get an expression for the frequency of the harmonics as a function of the tension:

$$f = \frac{n}{2L}\sqrt{\frac{T}{\mu}} \tag{11}$$

This equation will be modeled in Part C of the experiment.

III. Experiment

SAFETY WARNING
Stay clear of the motor and vibrating shaft while the motor is on. Remove or secure any loose fitting sleeves or obtrusive jewelry, and tie back any hair from your face.

Part A: Getting started

(1) Open the Excel spreadsheet template for **Lab 8** found the Lab Templates folder on your lab station computer.

(2) Fill in your name and your lab partner's name, and choose your lab section number. *Failure to provide the correct section may result in grading problems with your report; please ask your TA if you aren't sure of your section number.*

(3) **Take a picture** of you and your partner with the lab apparatus by pointing the provide web cam appropriately and clicking "Capture image" in your lab template. If an error occurs, please consult your TA for assistance.

(4) To find the mass per unit length of the string, your TA will have already measured the length of a separate piece of string and then used a precision electronic balance to find the weight. The mass m and length z should be recorded on the board for you. Record these two values in the appropriate cells on your spreadsheet, and use them to calculate the mass per unit length of the sample string in SI units using $\mu = m/z$.

Part B: Frequencies of the fundamental and higher harmonics

The goal of this part is to measure the frequency and wavelength of the first four harmonics of a standing wave on a string and use them to determine the wave speed. The waves are produced by running one end of the string across a shaft with an eccentric cross-section connected to a motor; the rotation of the shaft causes the string to vibrate with small amplitude (see Figure 2). The frequency of the wave is controlled by varying the speed of the motor.

(1) Place a 50g weight on the weight holder; measure the ***total mass M of the weight and holder,*** and record the value in kilograms in your spreadsheet in the designated cell in under Part B.

(2) Check that the motor is firmly held in place and will not slide around, then hang the holder with the 50g weight on the end of the string opposite the motor, which should hang over the side of the table using a clamped pulley. **Take a moment to carefully stop the hangar from swinging; ANY motion there will increase the tension in the string in pulses due to centripetal force.**

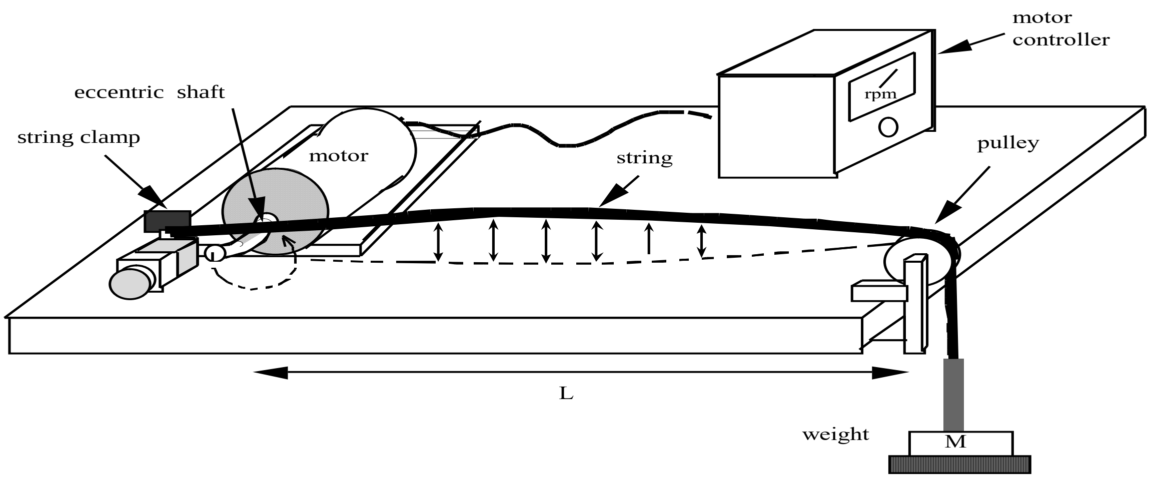

Figure 2. The weight applies tension $T = mg$ to the string, and the eccentric shaft on the motor pushes against the string, causing it to vibrate as the motor turns. The string is shown oscillating in its fundamental mode.

(3) Measure the length of the string L between the top of the shaft and the top of the pulley in meters and record your value in the spreadsheet.

(4) Examine the ruler and the setup and estimate the uncertainty ΔL in your measurement of L; record the value in your spreadsheet.

(5) Turn on the motor, and starting with $f = 0$, increase f gradually until you find the point of resonance with the fundamental harmonic; you will know you are there when the amplitude of the wave increases substantially. Be sure that your mode has the right n = 1 shape—the $n = 1$ mode has an anti-node in the middle of the string and the only nodes at the ends **(see Figure 2)**. Note that the frequency is high enough that you will see the **string blurred** to fill the entire elongated almond shape between +A and –A.

The wave amplitude at resonance should be about 1-2 cm. To check that you are precisely on resonance, try increasing and decreasing the frequency by extremely small amounts while keeping an eye on the amplitude in the center, which should **decrease** when you move a bit ***either above or below*** the resonant frequency. It can be tricky to find the fundamental the first time because the range of frequencies for which the amplitude is large is rather narrow, and it takes a few seconds for it to react and settle down into a steady vibration. Patience and double-checking are key.

Record the frequency f of the fundamental mode in your spreadsheet in the designated cell. Leave the value in rpm (revolutions / minute) for now, and record the units as well.

(6) Look at the scale on the motor controller and estimate the uncertainty in your measured frequency Δf. The rule of thumb for this type of scale is to take the uncertainty as **1/3 of the smallest increment** displayed on the scale (see Appendix A). Record Δf in your spreadsheet, also in rpm.

(7) Measure the node-to-node separation *y* and record the value in your spreadsheet. For the fundamental mode, this should simply be equal to the length of the string *L*. For other modes, you can measure the distance between any two adjacent nodes **except the one at the shaft**. **Do not place the ruler on the moving shaft as you will dissipate the wave and expose yourself to potential hazards.**

The precise location of a node is considerably easier to see from the side than from above; however, using the tape measure is much more easily done from above. To mitigate this issue, a **small simple wooden pointer** is provided for your convenience; if needed, place the pointer at the node location while viewing from the side, then measure to the pointer location from above.

(8) Begin increasing the frequency in search of the next harmonic, and repeat steps 5-7 for n = 2, Do the same for n = 3 and n = 4 afterward, being careful to check that the weight hangar is steady each time. Make sure that the modes you measure are shaped like those shown in Figure 1.

Part C: Dependence of the fundamental frequency on string Tension

The main goal of this part is to measure how the fundamental frequency varies with the tension in the string. You will be able to vary the tension by adding additional weights to the weight hangar.

(1) Remove the hangar from the end of the string and remove the 50g weight. Measure the mass of the empty hangar and record it as the first entry (after units) in the column labeled "M" in Part C of your template. It should also have a mass of about 50g (0.050 kg).

(2) Place the hangar back on the end of the string and make sure to again **steady it** as **completely** as possible. Then repeat the process from step 5 in Part B to find the frequency of the fundamental mode. Record its value and uncertainty in your spreadsheet in the designated cells in Part C.

(3) Add a 25g mass to the hangar, re-steady it, and then find the new frequency of the fundamental mode. Record the total mass (75g) and the frequency in the designated areas of Part C.

(4) Continue adding mass to the hangar and find the frequency of the fundamental mode each time for total masses of about 100, 125, 150, 200, 250, 300, 350, and 400g. You already measured the fundamental mode for 100g in part B, so you can simply enter that same frequency into the appropriate cell in part C.

IV. Analysis

In this part you will use the data from the previous sections to test the validity of the models you measured when compared to the theoretical expectations.

Part B: Frequencies of the fundamental and higher harmonics

(1) In the designated cell in Part B of your spreadsheet, compute the tension force $T = Mg$ using $g = 9.801$ m/s^2 (tension in Newtons).

(2) Convert your measured harmonic frequencies f from rpm to Hz by dividing your recorded rpm reading by 60. Also convert your uncertainties from rpm to Hz.

(3) Calculate the wavelength λ for each mode as 2x the node-to-node spacing.

(4) Calculate the wave speed v for each mode using $v = \lambda f$.

(5) Make a scatter plot of frequency f in Hz versus the mode number n. Be sure to label the axes, add units to the f axis, and put a title on the plot.

(6) Add Excel's linear trendline to your plot. Be sure to check the box to display the equation.

(7) Finally run the "f versus n" macro to do a linear χ^2 fit to your data and evaluate the validity of the fit. Check that the slope and intercept are nearly the same that Excel found for its trendline.

Final Question 1: *Does your data for the harmonics show the expected linear relationship to the mode number? Justify your answer with a brief statistical explanation as well as a comment on the visual validity of the fit in your plot.*

Part C: Dependence of the fundamental frequency on string Tension

(1) For each mass, compute the tension $T = Mg$ in the designated column in Part C of your spreadsheet.

(2) Convert your measured mode frequencies f from rpm to Hz by dividing your recorded rpm reading by 60. Also convert your uncertainties to Hz.

(3) For each mass, use the frequency f to find the wave speed v using $v = \lambda f$. In this case, recall you were always measuring the fundamental mode, so $\lambda = 2L$ and the wave speed is simply

$$v = 2 f L \tag{12}$$

(4) For each calculated v propagate the uncertainty in velocity Δv for each measurement using

$$\Delta v = v\sqrt{\left(\frac{\Delta L}{L}\right)^2 + \left(\frac{\Delta f}{f}\right)^2} \quad . \tag{13}$$

(5) Plot your measured velocity v versus tension T, and add Δv error bars, axes labels, and a title to your plot.

(6) Compute the theoretically expected wave speed v from the tension T and mass density m for each tension using equation (10) in the designated column.

(7) Add a plot of v_{theory} versus T to your plot you can compare the theory to your v versus T data.

(8) Add Excel's power law trendline to your plot. Be sure to check the box to display the equation.

(9) Since the Excel fit does not give you the uncertainties in the parameters, run the "Best v versus T" macro to minimize χ^2 and find the best fit of your data to a function of the form

$$v_{\text{fit}} = \left(\frac{T}{\mu_{\text{fit}}}\right)^n$$

You should find that the exponent n ~ 0.5 and closely matches the value from Excel's trendline, and also that μ_{fit} is close to the value for μ we calculated at the start of the experiment.

Final Question 2: *Do your results show that the wave speed is independent of frequency? Identify any inconsistencies and give a statistical argument to support your conclusion.*

Final Question 3: *Do your data and fit for v vs T agree with theory ? Justify your answer with a brief statistical explanation, and additionally compare the values for v that you calculated using equation (12) for agreement with the theory values. Be sure to take uncertainties into account.*

V. Finishing Up Before Leaving the Lab

(1) Record your answers to the Final Questions in your Lab spreadsheet. Yes or no questions should be justified or explained adequately.

(2) Check over your spreadsheet to make sure that you have completed everything, and that you have not missed any steps or left red feedback messages unaddressed. The automatic feedback system on the template has limited ability to detect problems, so check carefully, and consult the TA if you think your work is incorrect.

(3) Save your spreadsheet using the provided button and submit your spreadsheet on ELMS before you leave. Both partners should do this.

(4) Log out of ELMS when you are done, but **do NOT log out on the computer**, just leave it at the desktop.

Each student needs to submit a copy of their spreadsheet to their own account on ELMS before leaving the lab.