Experiment V — The Conical Pendulum

I. Preparing for Lab

The purpose of this experiment is to test a model of the conical pendulum, which is an interesting and unique example of centripetal motion.

To prepare for this lab before your session starts, read through the Physical Theory section below; for further reference, see section 6.2 on centripetal motion and sections 16.2 – 16.8 on oscillations in your textbook.

If you wish to review it, a video walkthrough of a similar experimental setup is available here.

*Finally, you must complete the **Pre-Lab questions** on **Expert TA** before your lab starts.*

Equipment:

Centripetal motion apparatus with fixed optical gate
Removable 1-cm x 1-cm grid with green screen on reverse

1.5 m ruler	Vernier calipers
Motor controller	Lab Pro Interface
Logitech web camera	Excel spreadsheet template

Logger Pro template for Conical Pendulum

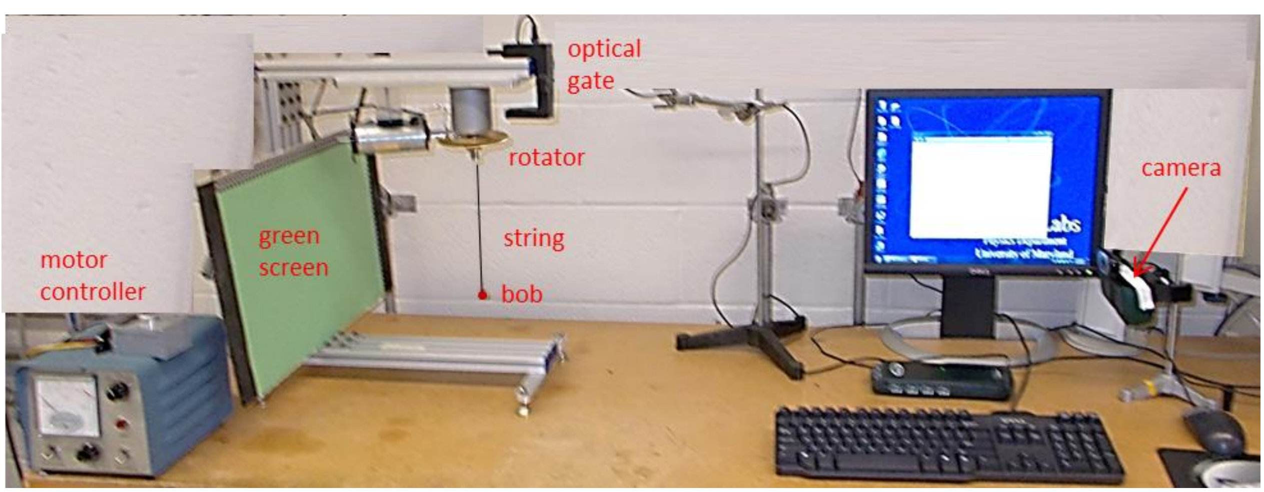


Figure 1. Photograph of prototype of centripetal motion apparatus, motor controller and camera.

II. Physical Theory

Review of Circular Motion

A body is accelerating whenever the magnitude ***or*** the *direction* of its velocity vector $\vec{v}$ is changing. When an object moves in a circular path at a constant speed *v*, the velocity is continuously changing in direction despite the constant magnitude. This type of acceleration is called **centripetal acceleration**. The magnitude of a centripetal acceleration is determined by the size of the circle (via radius *r*) as well as the speed of the motion

$$|\vec{a}_c| = \left|\frac{d\vec{v}}{dt}\right| = \frac{v^2}{r} \qquad (1)$$

where *v* is the linear speed; since the motion is circular and periodic, the speed can be understood as

$$v = \frac{2\pi r}{\tau} \qquad (2)$$

where $2\pi r$ is the circumference of the circle and τ is the period, i.e. the time it takes to complete one revolution. Substituting Equation (2) into (1) gives:

$$|\vec{a}_c| = \frac{4\pi^2 r}{\tau^2} \qquad (3)$$

The direction of the centripetal acceleration vector $\vec{a}_c$ is **always** towards the center of the circle the object moves in, which is always changing; this can be complicated to state in a Cartesian coordinate system, but in terms of the polar unit vector $\hat{r}$, it can be described generally as

$$\vec{a}_c = |a_c|(-\hat{r}) \qquad (4)$$

where $\hat{r}$ points from the center of the circle towards the mass, so that $-\hat{r}$ points inward.

As is the case with any acceleration, according to Newton's 2nd law, $\vec{a}$ is the result of the presence of a net force on the object;

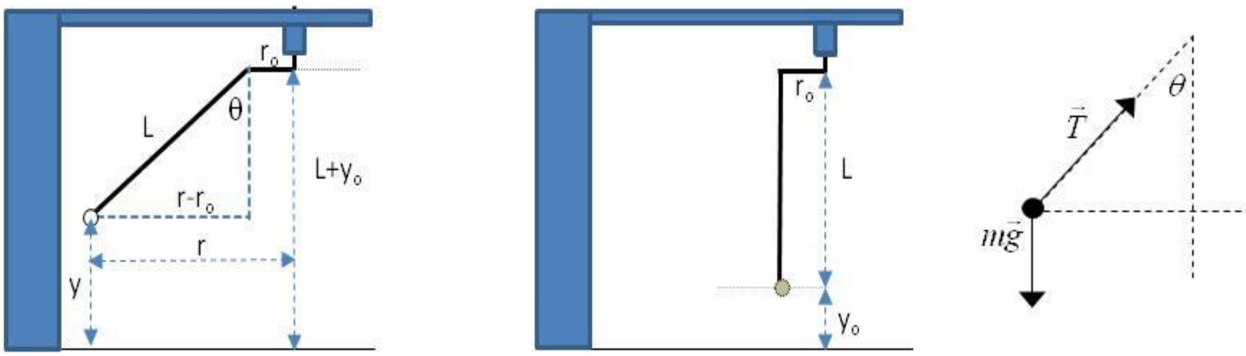
$$\vec{F}_{net} \equiv \sum_i \vec{F}_i = m\vec{a}$$

In the case of centripetal motion, that net force must also continuously point toward the center of the circle; we call this net force a centripetal force

$$\vec{F}_c = -(ma_c)\hat{r} \qquad (5)$$

Relationship between the period τ and the radius r in a conical pendulum

In this experiment a spherical mass *m* is suspended from a light string and is driven to swing in a horizontal circular path. This arrangement is called a **conical pendulum**. Figure 2(a) shows the geometry of the mass when it is moving, while Figure 2(b) shows the geometry when it is



(a) (b) (c)

Figure 2. (a) Geometry of the pendulum at an instant when the ball is smoothly revolving about the central axis on the string of length *L*. (b) Geometry of the pendulum when the ball is stationary; note the offset distance r_0 from the center of rotation. (c) Free body diagram for the mass *m* during motion.

hanging stationary in equilibrium. *L* is the length of the string, θ the angle the string makes with the vertical, and *r* is the radius of the circular orbit. In order to promote stable circular motion, the string is attached to the rotor at a small offset distance r_0 from the center.

Let *y* be the vertical coordinate of the bob when the pendulum is revolving, and y_0 be the vertical coordinate of the center of the mass when the mass is at rest and $\theta = 0$.

When the mass is revolving steadily, it is in a *dynamic equilibrium*, with the centripetal acceleration constant in magnitude and always pointing toward the center of the path; since the circular path is in the horizontal place, a_c is in the horizontal direction only, while $a_y = 0$.

Figure 2(c) shows the free-body diagram of the dynamic equilibrium; the only forces on the mass are due to gravity and the tension *T* in the string. Examining the diagram, one can see that in the vertical direction Newton's second law gives

$$mg = T\cos\theta \tag{6}$$

while in the horizontal direction

$$T\sin\theta = ma_c \tag{7}$$

Our conical pendulum apparatus is designed to easily allow for measuring the period of the motion τ and the radius of the circular path *r* (actually the diameter). Therefore we would like to use the 2^{nd} law equations (6) and (7) with the centripetal acceleration magnitude expression in equation (3) to derive a model equation for $\boldsymbol{\tau(r)}$. Dividing Equation (7) by Equation (6) gives

$$\frac{a_c}{g} = \tan\theta \tag{8}$$

With a little effort, one can find from Figure 2(a) that

$$\tan\theta = \frac{r - r_0}{\sqrt{L^2 - (r - r_0)^2}} \tag{9}$$

Substituting equation (3) for a_c and equation (9) for $\tan\theta$ into equation (8), one can re-arrange to find an expression for period τ in terms of orbit radius r and string length L

$$\tau = 2\pi\sqrt{\frac{r\sqrt{L^2-(r-r_o)^2}}{g(r-r_o)}} \qquad (10)$$

This is clearly a complex equation, but notice a few important characteristics:

- A little simplification shows that the dimensions under the radical are still L/g, which is the same as for a simple pendulum harmonic oscillator
- Also like the simple pendulum, the period does not depend on the mass of the bob.
- The expression would be much simpler if the small offset length r_0 was zero, but must be non-zero so that the motor can apply torque to the mass to create a stable orbit and bring the system into dynamic equilibrium.
- The minimum radius of the orbit is $r = r_0$ at which the period goes to infinity, and the maximum radius of the orbit is $r = r_0 + L$ at which the period would go to zero (equivalent to infinite frequency).

III. Experiment

SAFETY WARNING!

This apparatus has exposed rotating gears in the rotor above the pendulum. These rotating components can cause injury through entanglement with hair or clothing. Stay clear of the gears at all times. Remove or secure any loose fitting sleeves or obtrusive jewelry, and tie back any hair from your face.

Part A: Getting started and Setting up the Camera

(1) Open the Excel spreadsheet template for **Lab 5** found the Lab Templates folder on your lab station computer.

(2) Fill in your name and your lab partner's name, and choose your lab section number.
Failure to provide the correct section may result in grading problems with your report; please ask your TA if you aren't sure of your section number.

(3) In this lab, we will use the web cam for data collection, so **you won't take the usual selfie** today. To enable quantitative analysis of data images, the camera needs to be **in the same location for all of the images** you acquire. You should find it is near the left edge of the work desk, about 60-70cm to the right of the front edge of the apparatus (see Figure 1). ***If it is not at least 55-60cm away, rearrange the set up so you can move it back to this distance.***

Adjust the camera height so that it is at about even with the middle of the pendulum string. Then go to the computer desktop and click on the Logitech camera icon. In the app click

"Quick Capture", and you should see a live video feed and the camera controls. If you don't see them, click the "Controls" button to open them. Check that the camera is set with the autofocus off and resolution at 720.

(4) Place the large board with the black and white screen **in the holder on the apparatus base directly under the pendulum**, nearer to the front (camera end). Carefully adjust the height, angle, and alignment of the camera as well as the ZOOM control in Logitech until you see

- just the tip of the pendulum support at the top of the picture and
- about 5 boxes of space showing below the bob, as seen in Figure 3 below.
- the bob is centered horizontally in the grid
- the grid is square to the camera

NOTE: You will have to manually adjust the height and angle of the camera to get the desired orientation. Do not use the 4 green arrow Logitech controls to pan or tilt the camera, which will cause errors in the image capture algorithm.

To undo any adjustments to tilt or pan, click the circular button in the center of the arrows to re-center the camera.

(5) If needed, adjust the focus of the camera to ensure that the grid is fairly sharp.

(6) **Close the Logitech software,** and in your spreadsheet, click "Grid Background" to capture your image like the above. The macro uses a MatLab script to take the picture and import it into your template. If you get an error message, click "End" and try again. If it still does not work, notify your TA.

Once you are satisfied with your grid background image, do not touch the camera for the rest of the lab, or the size scale on your images will not be accurate.

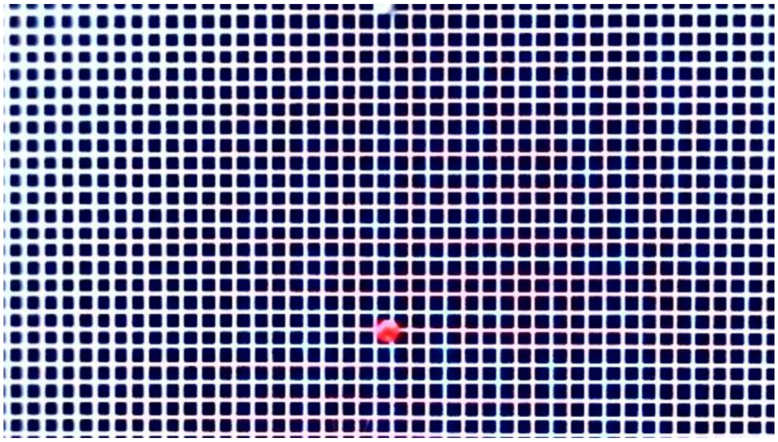


Figure 3. Sample picture of pendulum centered in background grid. The tip of the pendulum support is just visible at the top of the screen, and the bob is about 5 boxes above the bottom of the image. The bob is horizontally centered. The grid lines are fairly square.

If you need to revisit the Logitech controls at any point, close them again before taking any images in Excel, as Matlab and Logitech can control the camera only one at a time.

(7) Remove the grid board, turn it around so that the green side is facing the camera, and place it **in the holder on the base at the back of the apparatus**, farthest from the camera. This background will serve as a "green screen" on which to overlay the grid images for measuring the pendulum path diameter.

(8) Measure the distance H from the camera to the **center** of the pendulum rotor, and record the value in your template in cm. *For this lab, all your length measurements will be in cm.*

(9) Measure the length L of the pendulum from the top of the string to the middle of the bob; record L and an estimate of the uncertainty ΔL in your spreadsheet.

(10) Notice that offset length r_0 (see Figure 2) is given in the template.

Part B: Measuring the Period and obtaining images of the motion

The goal of this part of the experiment is to measure the period of the bob in motion at eight rotation frequencies and collect an image of each case.

(1) Make sure that the green screen is in place and everything is out of the way of the pendulum.

(2) On your computer desktop click on the "Logger Pro Templates" folder and open the "Conical Pendulum" file.

(3) Turn the motor controller power on and adjust the potentiometer on the small box (see below) to set the motor rotation frequency to 3000 rpm.

Note that 3000 rpm (50Hz) will NOT be the rotation rate of the pendulum because the large gear wheel at the top of the apparatus reduces the rate by a factor of about 27, so the period of the bob rotation will be only about 1.8 Hz.

Figure 4. Motor Speed Control; use the knob on the small attached box, not on the motor.

(4) The rotating gear will of course turn as fast as it is being driven, **but it will take several minutes for the bob to "catch up" with the gear** and reach dynamic equilibrium at this drive frequency. More importantly,

- **It is *essential* that your bob reach the *same rotation rate* as the gear before you take the picture, in order for your data to be *even usable* later.**
- **The only way to see if the rate is correct is to watch the "keyhole" in the gear (located in the same direction as the string offset) and the bob itself, *both at the same time, carefully and for several periods,* until you can visually confirm the motions are synchronized.**
- **There will be several "fake equilibria" during which the bob will semi-stably rotate in resonance with the gear but NOT in unison with it, at rates like 5-to-3 and 3-to-2.**

Hence this is a rather difficult task that can be very taxing. The best solution *would be* to leave the apparatus alone and running for a "very long time" to ensure synchronization. That obviously is not feasible given the time constraints. So we suggest the following strategy.

(5) While you are waiting at first, click "Collect" in Logger Pro to start the collection of time data from the optical gate mounted in with the gear; this will measure the rotation period of the gear. The data should appear as points on the graph and as numbers in the three columns on the left of the screen.

(6) The collection of timing data should finish after 15 s. From the statistics pop-up window, enter the values of the mean period τ_{avg}, the standard deviation $\Delta\tau$, and the number of periods N into the 3000rpm section of part B section of your spreadsheet.

(7) When you believe the pendulum motion has stabilized, click the button labelled "3000 rpm" in your spreadsheet. The macro will use MatLab to take a series of pictures and stack the images together to create a single "blurred" image of the trajectory of the bob, with fairly sharp outer limits during which the bob is moving primarily perpendicular to the image plane. **Do NOT stop the bob rotation now or until after all images are collected.**

Note also that the routine has overlaid your background grid image, which now provides a faithful 1cm x 1cm grid in the plane cutting the pendulum cone in half; you can therefore use this grid to measure the diameter of the orbit just by counting the boxes (from bob center on the left to bob center on the right).

(8) To check that your pendulum has actually synced with the rotation of the gear, we will proceed with the first measurement of the orbit of the bob and check that its value is reasonable. **This will avoid potential problems with imaginary values in your fit later!**

Carefully examine the output image for 3000rpm, and use the 1cm x 1cm grid to find the apparent diameter *D'* of the orbits, simply by counting the boxes (to at least the nearest half-box). Make sure to measure from the **center** of the ball on the extreme left side of the orbit to the center of the ball on the right (see Figure 5 below). If it is difficult to determine where the center of the mass is due to the blur, estimate the radius of the bob and approximate where the center would be. This box-counting can be quite tedious… if you need to enlarge the image to manage it, be sure to return the image to precisely the same size afterward.

If your bob was synced with the gear rotation, the diameter of the path *D'* should be *not much smaller* than double your string length, 2*L*. For example, if your string length is 16cm, then *D'* should be around 30cm. The angle θ is quite large (> 60^o) so this is reasonable. If your value is close to this expected range, then record the value in the table above the image and proceed to step 9.

If your value is considerably less than expected (e.g. < 29cm for a 16cm string), then your bob had not yet synced with the gear. By this point it has likely been long enough for sync to occur, so **delete your initial image and retake the picture**. Re-check the value of D' for the new image in the same manner, and repeat this process until sync is evident.

(9) Repeat steps (3) – (7) for the frequencies specified in your template. If you **adjust the frequency downward VERY slowly**, it will sync with the gear rather quickly… in only about 30 seconds or so.

At the lowest frequencies, the bob may never stabilize… if you find yourself waiting more than a couple of minutes for sync, then just proceed anyway.

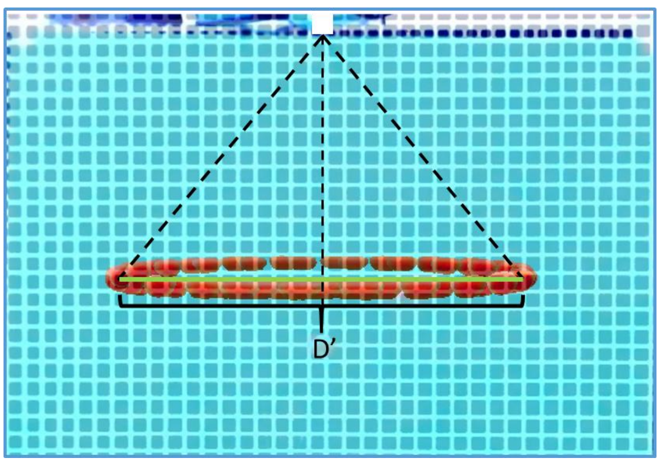


Figure 5. Image for a mid-range frequency with the cone geometry and the pertinent diameter *D'* superimposed. The number of boxes gives the length of *D'* (in real life) in cm.

IV. Analysis

In this part you will analyze the videos you took of the motion of the pendulum bob and compare your results to the theory.

(1) Carefully examine each remaining output image using the grid to find the apparent diameter D' of each of the orbits. Plug your values for D' for each orbit into the designated area of your spreadsheet in part B.

(2) Click the "Sort Data" button in the analysis section of the template. This macro will organize the data you collected into a table and put all of your images into one stacked comprehensive picture. ***Note this macro is extremely demanding for Excel and may fail for any number of reasons, if the image does not appear after a few tries, just notify your TA and proceed without it.***

(3) Calculate the uncertainties in the average period $\Delta\tau_{avg} = \Delta\tau / \sqrt{N}$ and the apparent radii of the orbits $r' = D'/2$ in the appropriate columns.

(4) Given that the camera is a distance H from the screen, there will be some **parallax** in your measurement of the apparent radius r' of the orbit. The "Correct for Parallax**"** macro will adjust your measured values of r' accordingly based on how far away the camera is from the pendulum and record the true values for r in the next column.

(5) Make a scatter plot of your eight measurements with τ on the y-axis and r on the x-axis. Label both axes, and add units and a chart title. As usual plot your data as points without any line connecting them. Finally, add error bars in the y-axis direction using your values for $\Delta\tau_{avg}$.

(6) Once all of your data is filled in and error bars are added, click "Fit Data" to run the least-squares regression macro and find best fit values for L, r_0, and g. Compare the fit values to the expected values. If you think you made a mistake in recording your data or uncertainties, you can go back and fix it, but make sure you rerun the macro, as it does not auto-update.

IMPORTANT: Since the equation for τ contains several square roots of differences, it is feasible for the fit to result in *imaginary results* if the data is bad enough. If you are getting "#NUM!" in one of more cells for your fit, it means your *D'* values were recorded for un-synced pendula. Consult your TA for how to proceed.

(7) Once you are satisfied with your fit results, add the τ_{fit} vs r data to your plot as a 2^{nd} set of points. Make sure the plot has a legend that clearly labels each set of points (i.e. data and fit).

(8) Input equation (10) to calculate the expected theoretical value for τ_{theory} as a function of the radius r in the designated area of your spreadsheet. Notice we have included a large number of radius values so that you can get a smooth curve for the theory. For this calculation you

will need to use your **measured** value for L, the given value for r_0, and $g = 980.1$ cm/s^2. Add the theory to your plot of τ vs r and reformat the theory points to be a line without markers.

Final Question 1: *Based on your fit to and plot of τ vs r, do your results agree with Eqn (10)? Give a statistical argument.*

Final Question 2: *Are your fitting parameter values for L and r0 consistent with the values found in part A? Comment on possible causes of any discrepancies.*

Final Question 3: *Calculate the percent difference between your fit value for g and the known value of $g = 980.1$ cm/s^2.*

V. Finishing Up Before Leaving the Lab

(1) **Record your answers to the Final Questions in your Lab spreadsheet. Yes or no questions should be justified or explained adequately.**

(2) Check over your spreadsheet to make sure that you have completed everything, and that you have not missed any steps or left red feedback messages unaddressed. The automatic feedback system on the template has limited ability to detect problems, so check carefully, and consult the TA if you think your work is incorrect.

(3) Save your spreadsheet using the provided button and submit your spreadsheet on ELMS before you leave. Both partners should do this.

(4) Log out of ELMS when you are done, but **do NOT log out on the computer**, just leave it at the desktop.

Each student needs to submit a copy of their spreadsheet to their own account on ELMS before leaving the lab.