Experiment

# VI — The Pendulum

## I. Preparing for Lab

The purpose of this experiment is to measure the dependence of the oscillation period of a simple pendulum on the pendulum's length and mass. You will attempt to validate your measurements by using them to calculate the acceleration due to gravity *g*.

To prepare for this lab before your session starts, read through the Physical Theory section below; for further reference, see the first half of Chapter 16 in your OpenStax textbook.

If you wish to review it, a video walkthrough of a similar experimental setup is available here.

*Finally, you must complete the **Pre-Lab questions** on **Expert TA** before your lab starts.*

### Equipment:

Variable length pendulum stand
A two-meter stick
Vernier calipers
LoggerPro template for the plane pendulum
Wood and metal bobs of different mass
Photogate timer
Excel Spreadsheet template

## II. Physical Theory

Consider an object hanging from a string in equilibrium. At this point, the tension in the string (vertically upward) and the force of gravity on the object (vertically downward) are balanced. If a sideways force disturbs the object, it will swing outward, it will continue for some time to swing in oscillation, forming a simple pendulum.

Consider this pendulum, with mass *m* and length *L,* while it is swung out to an angle θ from the vertical (you can think about it swing to the left or the right due to the symmetry). At this angle, the tension in the string (now along the string at angle θ) and the force of gravity no longer be in balance. One can show that the gravitational force will exert a torque $\vec{\tau}$ on the pendulum bob about the pivot point at the top of the string with magnitude $|\vec{\tau}| = (mg\sin\theta)L$. If the angle is << 1 radian (1 radian is about 57°, so say θ = 10° or less), then the small angle approximation $\sin\theta \approx \theta$ will hold and we can write the torque as

$$\tau = mgL\theta \tag{1}$$

Recall that the rotational form for Newton's Second Law is

$$\vec{\tau}_{net} = I\vec{\alpha}\ , \tag{2}$$

where $I$ is the moment of inertia of the pendulum and

$$\vec{\alpha} = \frac{d^2\theta}{dt^2}$$

is the angular acceleration. Treating the bob as a point mass, its moment of inertia is

$$I = mL^2 \tag{3}$$

Combining Equations (1-3) and simplifying, one finds

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\theta \tag{4}$$

where the negative sign indicates that θ is shrinking when $\alpha$ is growing and vice versa.

This linear second order differential equation has the form of a harmonic oscillator, so the general solution can be written as:

$$\theta = \theta_0 \cos(\omega t + \phi_0), \tag{5}$$

where $\theta_0$ is the maximum angle the pendulum reaches during its oscillation, $\omega$ is the angular frequency of oscillation of the pendulum, and $\phi_0$ is the phase angle of the pendulum when $t = 0$.

Recall that from equation (4), the magnitude of coefficient of θ on the right-hand side is equivalent to the $\omega^2$ so that

$$\omega = \sqrt{\frac{g}{L}}\ ,$$

indicating that the frequency of the oscillations is dependent on these properties of the system. So then the period $T$ of the oscillations is

$$T = \frac{2\pi}{\omega} \qquad \Rightarrow \qquad T = 2\pi\sqrt{\frac{L}{g}} \tag{6}$$

Note that the period depends only on the length $L$ and gravitational field of the planet $g$ and is independent of the mass $m$. This implies that all simple pendulums of the same length oscillating at small angles will have the same period on the same planet, regardless of the characteristics of the mass that is swinging.

# III. Experiment

## Part A: Getting started

(1) Open the Excel spreadsheet template for **Lab 3** found the Lab Templates folder on your lab station computer.

(2) Fill in your name and your lab partner's name, and choose your lab section number. *Failure to provide the correct section may result in grading problems with your report; please ask your TA if you aren't sure of your section number.*

(3) **Take a picture** of you and your partner with the lab apparatus by pointing the provide web cam appropriately and clicking "Capture image" in your lab template. If an error occurs, please consult your TA for assistance.

(4) Make sure that the indicator light on the sonic ranger is glowing green. If it isn't, check that the cord for the ranger is plugged into the LabPro interface. If not, contact your TA.

(5) Start the LoggerPro software by opening the LoggerPro Templates folder on the desktop and clicking on the file labelled **Plane Pendulum**; if LoggerPro was left running from the previous lab section, close and restart it.

## Part B. Measuring the period of the metal bob

(1) Briefly note the difference in mass of the two bobs.

(2) Move the wooden bob out of the way, and adjust the position of the lower clamp on the pendulum stand so that the length of the string below the clamp is about 1m (3ft, you don't need a precise measurement yet). The **upper clamp** should not be moved during the course of the experiment. See figure 1b for details.

(3) Orient the clamps and the photogate so that

- The string is able to swing into the "C" of the gate and out again without hitting the gate or any other interference.
- The bob is fully below the photogate so that only the string cuts through the infrared beam path on the optical gate (you can see the little holes where it comes out and goes in, ***make sure the little door is open!***) See figure 1c for a clear picture of the bob and gate setup.

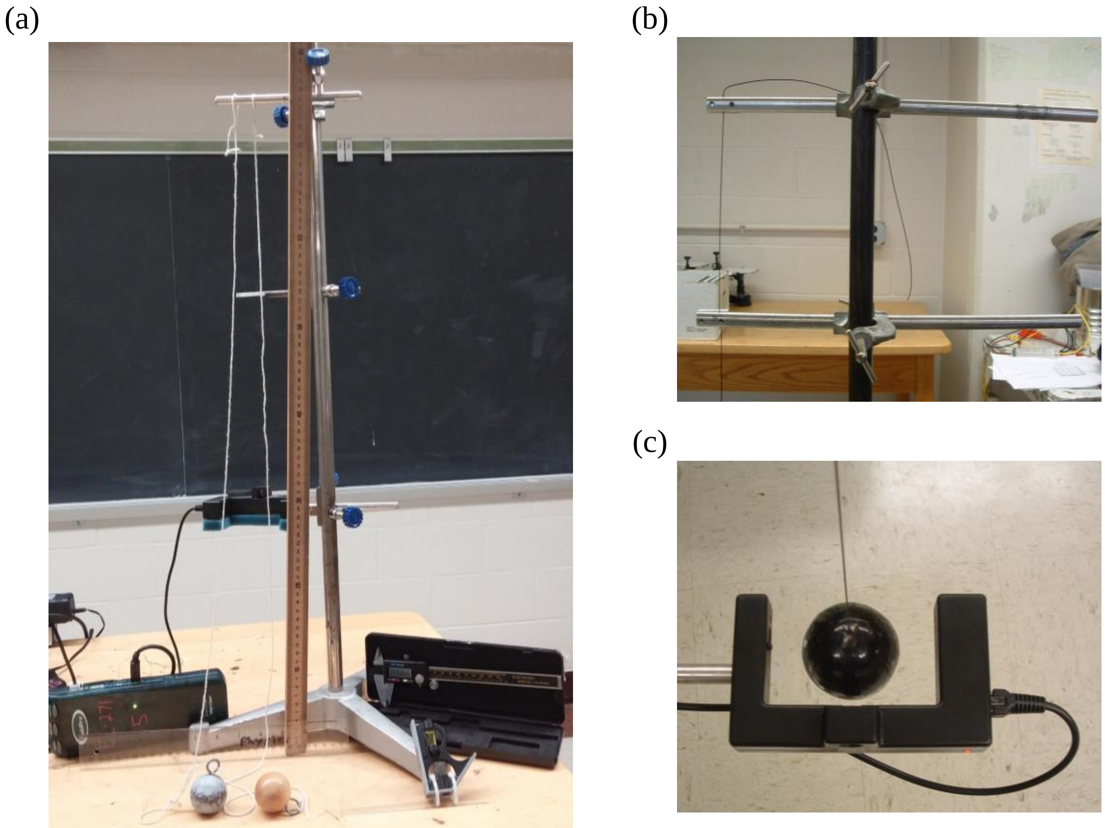


**Figure 1**. (a) Pendulum support stand with metal and wood bobs resting on the table, optical gate, and calipers. (b) The pendulum bob hangs from a string held by an upper support clamp and a lower support arm. (c) Pendulum string passing through photogate timer as it oscillates with a small amplitude.

(4) Use the meter stick to measure the length of the pendulum string, from the bottom clamp to the top of the bob, and record the length in meters under $y$ in your spreadsheet. Examine the meter stick to estimate the uncertainty $\Delta y$ in your measurement and record its value as well.

(5) Measure the diameter $D$ of the pendulum bob using the Vernier calipers, and record the measurement in meters; note that the metal bob is likely a bit dented and deformed, so try to roll it around and find a reasonably round cross section to measure.

(6) Set the pendulum **swinging so it travels a few centimeters at most** in the horizontal direction - remember that you need to keep the angle small for Equation (4) to hold. The bob should swing into the "C" of the gate and out again, so that the string passes through the beam path around the middle of the swing. Double check that the string is not colliding with the optical gate and make sure that only the string, and not the bob, is passing through the optical gate.

(7) Once the pendulum is swinging smoothly through the gate, go back to the LoggerPro template and click on the "Collect" button to set the data collection in motion. Data will be collected for 20s.

(8) When collection is complete, go to the "Analyze" menu and select "Statistics"; set the format to **display 4 decimal places** - *you will need all those digits*. Then enter into your Excel spreadsheet the values for the average period $T_{avg}$, the standard deviation of the periods $\sigma_T$, and the number of periods $N$ reported in the statistics display.

(9) Given that the average period $T_{avg}$ was calculated from $N$ measurements of $T$ with a standard deviation of $\sigma_T$, the **uncertainty** $\Delta T_{avg}$ will be

$$\Delta T_{avg} = \sigma_T / \sqrt{N} \ . \tag{7}$$

Use EXCEL to calculate this in the designated cell.

(10) Repeat 2-4 and 6-9 for three additional pendulums with lengths of about 0.85m, 0.7m, and 0.55m.

### Part C. Measuring the period of the wooden bob

*The purpose of this part is to measure the oscillation period of a second bob with different mass in an attempt to verify that the oscillation period is independent of the mass.*

(1) Move the metal bob out of the way and replace it with the wooden bob.

(2) With the wooden bob, repeat steps 2-9 from Part B for pendulums of about the same four lengths (they don't need to be exactly the same). Remember to use the calipers to measure the diameter of the wooden bob and record it in your template.

*The template's auto-feedback is turned off for this part, so be careful to avoid typos.*

## IV. Analysis

(1) In the designated columns of Parts B and C in your spreadsheet, compute the pendulum length for each measurement by adding the string length to the ***radius*** of the bob:

$$L = y + D/2 \tag{8}$$

Also record the uncertainty $\Delta L$, which we will approximate as **equal to $\Delta y$**, since the bob diameter measurement should have an uncertainty which is significantly smaller. The uncertainty will be the same for every length, so fill in the same value for all the cells in the column.

(2) Make scatter plots of your $T_{avg}$ vs $L$ data for both bobs. Add $y$-error bars $\Delta T_{avg}$ and $x$-error bars $\Delta L$ to your plots.

(3) On your metal bob graph, add a **power-law** trendline to your plot by right-clicking on one of the data points, selecting **add trendline**, and then in the window at right, selecting **Power** from the list. Be sure to also click on the box to display the equation on the chart.

(4) The fit macro does a $\chi^2$-fit of your $T$ vs $L$ data with a function of the form $T = aL^b$. Click the macro button for the metal bob to get the best fit values for $a$ and $b$. According to the Equation (4), you should expect to find $a = 2\pi/\sqrt{g}$ and the exponent equivalent to a square root, $b = 0.5$.

The macro also reports $\chi^2$, the number of fit degrees of freedom (see Appendix A), and whether or not your fit is reasonable. Unlike the power law trendline fit that Excel did, *this macro takes into account the uncertainty in each point* and therefore should give a better fit.

(5) Check whether your fit for $b$ is statistically consistent with ½ based on the value and its uncertainty; report your answer in the box next to your plot.

(6) Using the best fit parameters $a$ and $b$ and the $L_{\mathrm{fit}}$ data provided, input the formula $T_{\mathrm{fit}} = aL^b$, in the appropriate column of your spreadsheet. Add this curve data to your plot as a smooth curve with no points.

Your metal bob plot should now have your original data (as points), the Excel trendline, and the best fit curve. **Add a legend** to the plot to differentiate the three.

(7) From the best fit value for the parameter $a$, use input a formula to determine the acceleration due to gravity given using

$$g = 4\pi^2 / a^2 . \qquad (8)$$

Since $g$ is calculated using only the one parameter $a$, the propagation of the uncertainty $\Delta g$ contains only one term, but the quadratic relationship gives a factor of the two between the relative uncertainties:

$$\Delta g = \sqrt{\left(\frac{\partial g}{\partial a}\Delta a\right)^2} = \sqrt{\left(-2\left(\frac{4\pi^2}{a^3}\right)\Delta a\right)^2} = \sqrt{\left(-\frac{2g}{a}\Delta a\right)^2} = \frac{2g}{a}\Delta a \qquad (10)$$

Use the resulting expression to calculate $\Delta g$ in the designated cell.

***Final Question 1:*** *The accepted value for g on the third floor of the physics building is 9.80102 m/s$^2$; this value was measured by the USGS and is recorded on a plaque down the hall. Does your result for g for the metal bob agree with the actual value? That is, did you find agreement with the accepted USGS value to within your estimated uncertainty?*

(8) Repeat steps 4-7 for your wooden bob data.

***Final Question 2:*** *Do your resulting values for g for the two pendulum bobs agree to within the experimental uncertainties?*

***Final Question 3:*** *Do your resulting values for the exponent b for the two pendulum bobs agree to within the experimental uncertainties?*

***Final Question 4:*** *When EXCEL did the power trend line fit to the metal bob data, did it find the same fit as the $\chi^2$-macro fit to within the uncertainties? If not, which fit is better?*

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## V. Finishing Up Before Leaving the Lab

**(1) Record your answers to the Final Questions in your Lab spreadsheet. Yes or no questions should be justified or explained adequately.**

(2) Check over your spreadsheet to make sure that you have completed everything, and that you have not missed any steps or left red feedback messages unaddressed. The automatic feedback system on the template has limited ability to detect problems, so check carefully, and consult the TA if you think your work is incorrect.

(3) Save your spreadsheet using the provided button and submit your spreadsheet on ELMS before you leave. Both partners should do this.

(4) Log out of ELMS when you are done, but **do NOT log out on the computer**, just leave it at the desktop.

**Each student needs to submit a copy of their spreadsheet to their own account on ELMS before leaving the lab.**