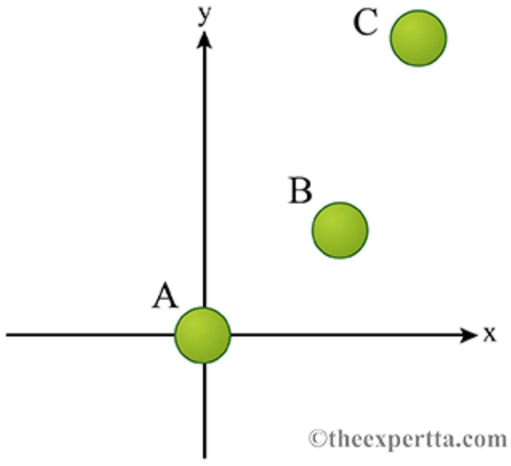
Introductory Physics Example Solutions

Problem 1 - 4.1.7 :

An object undergoing two-dimensional motion in the *xy* plane is shown in the figure as a motion diagram. The position of the object is shown after two equal time intervals of Δt each. The position at point A is (0,0), the position at point B is (x_B, y_B), and the position at point C is (x_C, y_C).



Part (a) **What is the average velocity of the object between position A and position B? Enter your answer as a vector in terms of the variables given above and the unit vectors i and j.**

The velocity components in each direction are treated separately. Each is determined by taking the difference, final value mius the initial value, of either the x coordinate for the i direction or the y coordinate for the j direction, respectively, and dividing by the elapsed time.

$$v_{AB} = \frac{(x_B - x_A)}{(\Delta t)}\mathbf{i} + \frac{(y_B - y_A)}{(\Delta t)}\mathbf{j}$$

Since point A has corrdinates (0,0), we write the average velocity as

$$\boxed{v_{AB} = \frac{x_B}{(\Delta t)}\mathbf{i} + \frac{y_B}{(\Delta t)}\mathbf{j}}$$

Part (b) **What is the average velocity of the object between position B and position C? Enter your answer as a vector in terms of the variables given above and the unit vectors i and j.**

The velocity components in each direction are treated separately. Each is determined by taking the difference, final value mius the initial value, of either the x coordinate for the i direction or the y coordinate for the j direction, respectively, and dividing by the elapsed time.

$$\boxed{v_{BC} = \frac{(x_C - x_B)}{(\Delta t)}\mathbf{i} + \frac{(y_C - y_B)}{(\Delta t)}\mathbf{j}}$$

Part (c) **Now assume that the velocity of the particle in each interval is constant, with values equal to the average velocity you found in parts (a) and (b). What is the average acceleration of the object over the entire interval shown in the figure? Enter your answer as a vector in terms of the variables given above and the unit vectors i and j.**

Acceleration is the change in velocity with time. Using the velocities determined for the two time intervals in parts (a) and (b), we find

$$a_{ave} = \frac{(v_{BC} - v_{AB})}{(2\Delta t)}$$

$$= \frac{\left(\left(\frac{(x_C - x_B)}{(\Delta t)}\mathbf{i} + \frac{(y_C - y_B)}{(\Delta t)}\mathbf{j}\right) - \left(\frac{x_B}{(\Delta t)}\mathbf{i} + \frac{y_B}{(\Delta t)}\mathbf{j}\right)\right)}{(2\Delta t)}$$

$$\boxed{a_{ave} = \frac{((x_C - 2x_B)\,\mathbf{i} + (y_C - 2y_B)\,\mathbf{j})}{\left(2(\Delta t)^2\right)}}$$

Part (d) **If x_B = *0.35* m, y_B = *0.31* m, x_C = *0.21* m, y_C = *1.7* m, and the time interval Δt = *0.6* s, what is the *x*-component of this acceleration, in meters per second squared?**

Use the x-component from part (c) and substitute the given values.

$$a_{ave,x} = \frac{(x_C - 2x_B)}{\left(2(\Delta t)^2\right)}$$

$$= \frac{((0.21\text{ m}) - 2(0.35\text{ m}))}{\left(2(0.6\text{ s})^2\right)}$$

$$\boxed{a_{ave,x} = -0.6806\text{ m/s}^2}$$

Part (e) **Using these same values, what is the *y*-component of the acceleration, in meters per second squared?**

Use the y-component from part (c) and substitute the given values.

$$a_{ave,y} = \frac{(y_C - 2y_B)}{\left(2(\Delta t)^2\right)}$$

$$= \frac{((1.7\text{ m}) - 2(0.31\text{ m}))}{\left(2(0.6\text{ s})^2\right)}$$

$$\boxed{a_{\text{ave,y}} = 1.5\text{ m/s}^2}$$

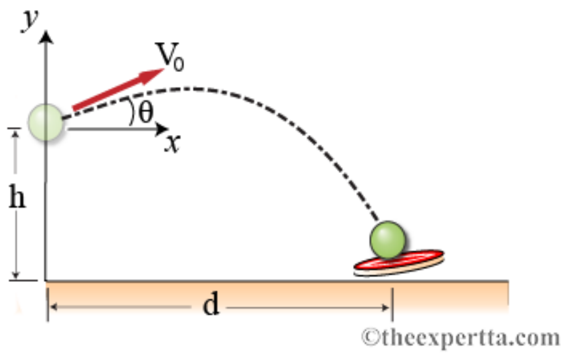
Part (f) **What quadrant is this acceleration in?**

The quadrants are numbered 1, 2, 3, 4, where quadrant 1 is the upper right where both x and y values are positive, then proceding counterclockwise. In the case of the x and y components for the acceleration in this problem, the x-component is negative and the y-component is positive. Therefore, the acceleration is in...

$$\boxed{\text{The second quadrant}}$$

Problem 2 - 4.3.6 :

A student throws a water balloon with speed v_0 from a height h = *1.5* m at an angle θ = *21*° above the horizontal toward a target on the ground. The target is located a horizontal distance d = *5.5* m from the student's feet. Assume that the balloon moves without air resistance. Use a Cartesian coordinate system with the origin at the balloon's initial position.



Part (a) **What is the position vector, R_{target}, that originates from the balloon's original position and terminates at the target? Put this in terms of *h* and *d*, and represent it as a vector using i and j.**

The tail of the vector drawn from the center of the balloon at the launch point to the center of the balloon at its final location is located at the coordinates $(0, h)$ and the head of the vector is at $(d, 0)$. To express the position vector in terms of these coordinates, we write the following using i, j notation.

$$\vec{R}_{\text{target}} = (x_f - x_i)\,\mathbf{i} + (y_f - y_i)\,\mathbf{j}$$

$$= (d - 0)\,\mathbf{i} + (0 - h)\,\mathbf{j}$$

$$\boxed{\vec{R}_{\text{target}} = d\mathbf{i} - h\mathbf{j}}$$

Part (b) **In terms of the variables in the problem, determine the time, *t*, after the launch it takes the balloon to reach the target. Your answer should *not* include *h*.**

From the equations of kinematics, we begin with

$$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$$

Since there is no acceleration, the final term in the equation is equal to zero. The first term is also equal to zero, since the balloon starts on the y-axis. To find the time in terms of the initial velocity and angle to reach $x = d$, we write:

$$d = v_{0x}t$$

$$t = \frac{d}{v_{0x}}$$

$$\boxed{t = \frac{d}{(v_0 cos\,(\theta))}}$$

Part (c) **Create an expression for the balloon's vertical position as a function of time, *y*(*t*), in terms of *t*, v_o, *g*, and *θ*.**

From the equations of kinematics, we begin with

$$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$$

Substituting in the variables associated with this particular problem and the acceleration due to gravity gives,

$$y = h + v_0 sin\,(\theta)\,t - 0.5gt^2$$

We are told in the problem statement that the origin is located at the balloon's starting position, therefore $h = 0$ in our equation.

$$\boxed{y = v_0 sin\,(\theta)\,t - 0.5gt^2}$$

Part (d) **Determine the magnitude of the balloon's initial velocity, v_0, in meters per second, by eliminating *t* from the previous two expressions.**

In part b, we found the time of flight

$$t = \frac{d}{(v_0 cos\,(\theta))}$$

In part c, we found the vertical position as a function of the time

$$y = v_0 sin\,(\theta)\,t - 0.5gt^2$$

By substituting the time of flight into the position equation, we can solve for the initial velocity of the balloon. Note that the balloon falls from $y = 0$, so its final vertical position is equal to $-h$.

$$h = v_0 \frac{sin\,(\theta)\,d}{(v_0 cos\,(\theta))} - 0.5g\left(\frac{d}{(v_0 cos\,(\theta))}\right)^2$$

which simplifies to

$$h = dtan\,(\theta) - 0.5g\left(\frac{d^2}{(v_0^2 cos^2\,(\theta))}\right)$$

Then,

$$h + dtan\,(\theta) = 0.5g\left(\frac{d^2}{(v_0^2 cos^2\,(\theta))}\right)$$

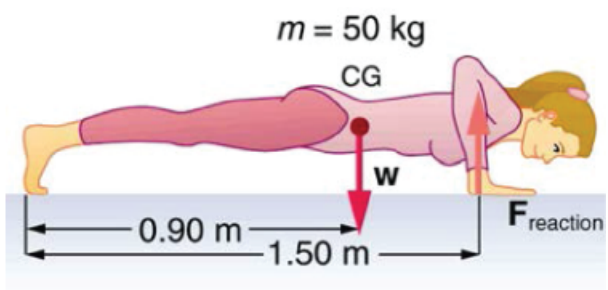
$$v_0^2 = 0.5g\left(\frac{d^2}{((h + dtan\,(\theta))\,cos^2\,(\theta))}\right)$$

$$\begin{aligned} v_0 &= \sqrt{0.5g\left(\frac{d^2}{((h+dtan(\theta))cos^2(\theta))}\right)} \\ &= \sqrt{0.5\left(9.81\text{ m/s}^2\right)\left(\frac{(5.5\text{ m})^2}{(((1.5\text{ m})+(5.5\text{ m})tan(21^\circ))cos^2(21^\circ))}\right)} \end{aligned}$$

$$\boxed{v_0 = 6.867\text{ m/s}}$$

Problem 3 - 7.5.26 :

Consider the forces involved in doing push-ups. The woman's weight acts as a force exerted downward on her center of gravity (marked CG in the figure). You may assume that the angle between her body and the floor is small enough so that the small angle approximation is valid throughout this motion.



Part (a) **Calculate the force, in newtons, the woman in the figure exerts to do a pushup at constant speed.**

In order for the woman to perform a successful pushup, her center of mass must move directly up without rotating. This means that the net torque about any point on her body must be zero. We can use the tips of her feet as the axis of rotation, giving us the following equation:

$$\tau_{\text{net}} = \tau_{\text{arms}} + \tau_{\text{CM}}$$

$$0 = F_r \cdot 1.50\text{ m} - F_g \cdot 0.90\text{ m}$$

$$F_g \cdot 0.90\text{ m} = F_r \cdot 1.50\text{ m}$$

$$mg \cdot 0.90\text{ m} = F_r \cdot 1.50\text{ m}$$

$$\frac{mg \cdot 0.90\text{ m}}{1.50\text{ m}} = F_r$$

$$F_r = \frac{}{1.50\text{ m}}$$

$$\boxed{F_r = 294\text{ N}}$$

Part (b) **How much work, in joules, does she do if her center of mass rises *0.23* m?**

While we know how far her center of mass raises, we do not know how far the woman stretches her arms. As such, we cannot attempt to solve this problem by multiplying the force her arms exert by the distance they are displaced. Instead, we can use the work-energy theorem in conjunction with the change in the woman's potential energy to solve for the work she must do. Letting the initial position of the woman's center of mass be our zero of potential energy, we get the following equation:

$$W = E_f - E_i$$

$$W = mgh - 0$$

$$W = mgh$$

$$W = 50\ \text{kg}\ \cdot 9.8\ \text{m/s}^2 \cdot 0.23\ \text{m}$$

$$\boxed{W = 112.7\ \text{J}}$$

Part (c) **What is her useful power output, in watts, if she does *21* push-ups in 1 min? Ignore any work done during the downward motion.**

To solve this problem, we can use the fact that power is equal to work divided by time. To begin, let's use our answer from part (b) to write an expression for the total work done if she does pushups for a minute.

$$W_{\text{total}} = W \cdot 21$$

$$W_{\text{total}} = 50\ \text{kg}\ \cdot 9.8\ \text{m/s}^2 \cdot 0.23\ \text{m}\ \cdot 21$$

Since we are considering a case where she does pushups for a minute, that means that the total time is equal to 60 seconds. We can therefore use the relationship between power, work, and time mentioned previously to write the following expression:

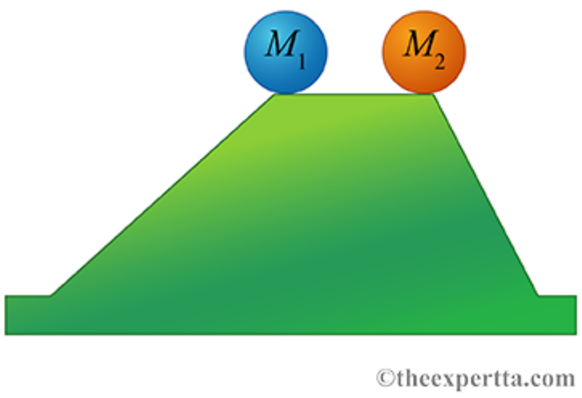
$$P = \frac{W_{\text{total}}}{t}$$

$$P = \frac{50\ \text{kg}\ \cdot 9.8\ \text{m/s}^2 \cdot 0.23\ \text{m}\ \cdot 21}{60\ \text{s}}$$

$$\boxed{P = 39.445\ \text{W}}$$

Problem 4 - c8.1.1 :

Two masses sit at the top of two frictionless inclined planes that have different angles, as shown in the figure.



Part (a) **What can be said about the speeds of the two masses at the bottom of their respective paths?**

The intial height is the same for both and they have the same gravitational potential energy. If both start from rest and finish at the same height at the bottom of the incline, then they will have the same kinetic energy, assuming that any frictional forces are negligible. When conservation of energy is applied to each ball,

$$M_1 gh = \tfrac{1}{2} M_1 v^2$$

$$M_2 gh = \tfrac{1}{2} M_2 v^2$$

In both cases, the masses cancel, resulting in the same value of the speed for both.

$$\boxed{\text{The two balls are traveling at the same speed.}}$$

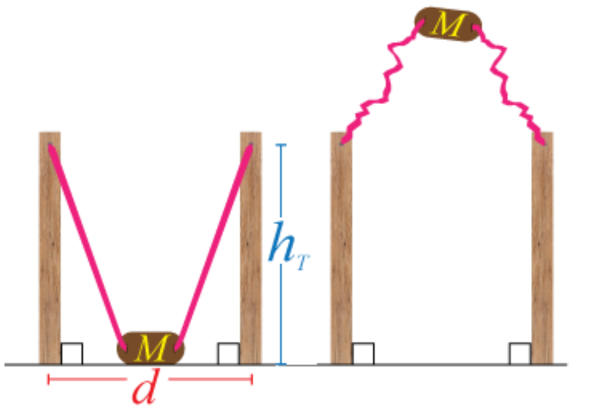
Part (b) **Which mass gets to the bottom first?**

In part a, we showed that the two balls have the same speed when they reach the bottom. However, mass 2 has a shorter distance to travel to the bottom, so it will arrive first.

$$\boxed{\text{Mass two.}}$$

Problem 5 - 8.3.11 :

The Slingshot is a ride for two people. It consists of a single passenger cage, two towers, and two elastic bands. Potential energy is stored in the elastic bands and the passenger cage is released. On the way up, this potential energy in the elastic bands is converted into the kinetic energy of the cage. At the maximum height of the ride, the energy has been converted into gravitational potential energy of the cage. The slingshot has two towers of height h = *71* m. The towers are a distance d = *31* m apart. Each elastic band has an unstretched length of L_0 = *39* m and a spring constant of k = *285* N/m. The total mass of the passengers and cage is m = *380* kg. The car is pulled down to the ground in the middle of the two towers.



Part (a) **Write an expression for the stretched length L of one of the elastic bands strung from the cage to the top of its tower.**

Using the left side of the drawing provided, we can see that we have been given two sides of a right triangle. To find the length L of the hypoteneuse, apply the Pythagorean theorem:

$$\boxed{L = \sqrt{\left(\frac{d}{2}\right)^2 + h^2}}$$

Part (b) **Calculate the stretched length, in meters, of each band.**

We saw in part (a) that the expression for the length is found using the Pythagorean theorem.

$$L = \sqrt{\left(\tfrac{d}{2}\right)^2 + h^2}$$
$$= \sqrt{\left(\tfrac{31}{2}\text{ m}\right)^2 + (71\text{ m})^2}$$

$$\boxed{L = 72.672\text{ m}}$$

Part (c) **Write an expression for the total potential energy stored in the elastic bands before the cage is released.**

The expression for the elastic potential energy is

$$U = \frac{1}{2}k\Delta x^2$$

where Δx is the distance that the elastic band has been stretched or compressed from its equilibrium position. In this problem, the distance $\Delta x = L - L_0$, where L was found in part (a).

Secondly, since there are two elastic bands, the total potential energy is twice the amount for a single band. Therefore,

$$U = 2\left(\frac{1}{2}\right)k\left(\sqrt{\left(\frac{d}{2}\right)^2 + h^2} - L_0\right)^2$$

$$\boxed{U = k\left(\sqrt{\left(\frac{d}{2}\right)^2 + h^2} - L_0\right)^2}$$

Part (d) **Calculate the total potential energy, in joules, stored in the elastic bands before the cage is released.**

$$U = k\left(\sqrt{\left(\tfrac{d}{2}\right)^2 + h^2} - L_0\right)^2$$
$$= (285\text{ N/m})\left(\sqrt{\left(\tfrac{31}{2}\text{ m}\right)^2 + (71\text{ m})^2} - 39\text{ m}\right)^2$$

$$\boxed{U = 323138.038\text{ J}}$$

Part (e) **Calculate the maximum height, in meters, of the ride.**

Assuming that energy is conserved, all of the elastic potential energy of the band becomes gravitational potential energy.

$$U = mgh = k\left(\sqrt{\left(\frac{d}{2}\right)^2 + h^2} - L_0\right)^2$$

$$h = \frac{k\left(\sqrt{\left(\frac{d}{2}\right)^2 + h^2} - L_0\right)^2}{mg}$$
$$= (285\text{ N/m})\frac{\left(\sqrt{\left(\frac{31}{2}\text{ m}\right)^2 + (71\text{ m})^2} - 39\text{ m}\right)^2}{(380\text{ kg})\left(9.80\text{ m/s}^2\right)}$$

$$\boxed{h = 86.772\text{ m}}$$

Problem 6 - 10.1.6 :

While sitting in physics class one day, you begin to ponder the workings of the analog clock on the classroom wall. You notice as the hands sweep in a continuous motion that there are exactly $t = 35$ minutes left in class.

Randomized Variables

$t = 35$ minutes

Part (a) **Through what angle (in radians) will the second hand turn before the end of class?**

Recall that there are 2π radians in a full rotation. Since the second hand completes a full rotation each minute, we can write the following equation for the angle:

$$\theta_{\text{seconds}} = 2 \cdot \pi \cdot 35$$

$$\boxed{\theta_{\text{seconds}} = 219.911 \text{ rad}}$$

Part (b) **Through what angle (in radians) will the minute hand sweep before the end of class?**

Recall that there are 2π radians in a full rotation. The minute hand will complete one rotation every 60 minutes, so we can consider one minute a 60th of a rotation. Based on this, we can write the following equation for the angle:

$$\theta_{\text{minute}} = 2 \cdot \pi \cdot \frac{35}{60}$$

$$\boxed{\theta_{\text{minute}} = 3.665 \text{ rad}}$$

Part (c) **Through what angle (in radians) will the hour hand sweep before the end of class?**

Recall that there are 2π radians in a full rotation. The hour hand will complete one rotation every 12 hours, and each minute is one 60th of an hour. Based on this, we can write the following equation for the angle:

$$\theta_{\text{hour}} = 2 \cdot \pi \cdot \frac{35}{12 \cdot 60}$$

$$\boxed{\theta_{\text{hour}} = 0.3054 \text{ rad}}$$

Part (d) **Calculate the angular velocity of the second hand in radians per second.**

To find the angular velocity, we can divide the angle of a full rotation in radians (2π radians) by the time it takes for a full rotation in seconds. We know that the second hand completes a rotation each minute, and that a minute is 60 seconds. The anguler velocity is therefore given by the following equation:

$$\omega_{\text{seconds}} = \frac{2\pi \text{ rad}}{60 \text{ s}}$$

$$\boxed{\omega_{\text{seconds}} = 0.1047 \text{ rad/s}}$$

Part (e) **Calculate the angular velocity of the minute hand in radians per second.**

To find the angular velocity, we can divide the angle of a full rotation in radians (2π radians) by the time it takes for a full rotation in seconds. We know that the minute hand completes a rotation each hour. Therefore, the first step is to convert one hour to seconds.

$$1 \text{ h} = 1 \text{ h} \cdot \frac{60 \text{ min}}{1 \text{ h}} \cdot \frac{60 \text{ s}}{1 \text{ min}}$$

$$1 \text{ h} = 3600 \text{ s}$$

Now that we know how long it takes the minute hand to complete a rotation in seconds, we can write the following equation for the angular velocity:

$$\omega_{\text{minutes}} = \frac{2\pi \text{ rad}}{3600 \text{ s}}$$

$$\boxed{\omega_{\text{minutes}} = 0.001745 \text{ rad/s}}$$

Part (f) **Calculate the angular velocity of the hour hand in radians per second.**

To find the angular velocity, we can divide the angle of a full rotation in radians (2π radians) by the time it takes for a full rotation in seconds. We know that the hour hand completes a rotation once every 12 hours. Therefore, the first step is to convert 12 hours to seconds.

$$12 \text{ h} = 12 \text{ h} \cdot \frac{60 \text{ min}}{1 \text{ h}} \cdot \frac{60 \text{ s}}{1 \text{ min}}$$

$12\ \text{h}\ = 43200\ \text{s}$

Now that we know how long it takes the hour hand to complete a rotation in seconds, we can write the following equation for the angular velocity:

$$\omega_{\text{hours}}\ = \frac{2\pi\ \text{rad}}{43200\ \text{s}}$$

$$\boxed{\omega_{\text{hours}}\ = 0.0001454\ \text{rad/s}}$$

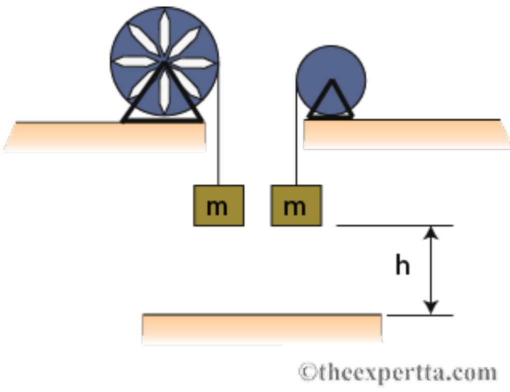
Part (g) **Is the angular acceleration of the second, minute, and hour hands the same? Pick the most accurate response.**

In order for a clock to be accurate, each unit of time (seconds, minutes, and hours) must be measured accurately. If the rate at which the hands turned changed, then you would run into things like one minute being longer than another. Since that clearly does not happen, we know that the clock must maintain a constant angular velocity for each hand with no angular acceleration on any of them. The correct answer is therefore:

Yes. The angular acceleration of each hand is zero.

Problem 7 - c10.4.1 :

Two identical masses are connected to two different flywheels that are initially stationary. Flywheel A is larger and has more mass, but has hexagonal sections where material has been removed. The attached masses are released from rest and allowed to fall a height *h*.



Part (a) **Which of the following statements about their angular accelerations is true?**

The two flywheels have differing moments of inertia in this situation. The tension in the two ropes is the same in the two cases. The tension causes a torque ($F = mg$) on each flywheel, which is also equal to the product of the moment of inertia and the angular acceleration.

$$\tau = rF = I\alpha$$

The moment of inertia for wheel A is

$$I_A = \gamma_A m_A r_A^2$$

and for wheel B,

$$I_B = \gamma_B m_B r_B^2$$

The product of the mass and radius is greater for wheel A than for wheel B. Also, the effect of the hexagonal holes in A makes $\gamma_A > \gamma_B$. Bring this all together,

$$\alpha_A = \frac{r_A F}{I_A} = \frac{F}{(\gamma_A m_A r_A)}$$

$$\alpha_B = \frac{r_B F}{I_B} = \frac{F}{(\gamma_B m_B r_B)}$$

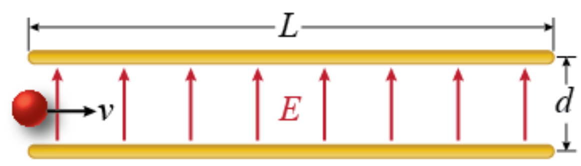
Therefore,

$$\alpha_B > \alpha_A$$

The angular acceleration of flywheel B is greater.

Problem 8 - 22.2.1 :

A particle with charge $q = +2$e and mass $m = 5.1 \times 10^{-26}$ kg is injected horizontally with speed 0.4×10^6 m/s into the region between two parallel horizontal plates. The plates are *22* cm long and an unknown distance *d* apart. The particle is injected midway between the top and bottom plates. The top plate is negatively charged and the bottom plate is positively charged, so that there is an upward-directed electric field between the plates, of magnitude $E = 31$ kN/C. Ignore the weight of the particle.



Part (a) **How long, in seconds, does it take for the particle to pass through the region between the plates?**

Near the central axis of a flat, charged conductor, the electric field can be well-approximated as pointing normal to the conductor's surface (parallel to the central axis). In the case of two oppositely-charged plates, the strength of the field remains a constant in this region, regardless of how close to either plate we look. We are concerned with the motion of a particle in just such a region.

Since the particle has a charge, it will have a force exerted on it from the charges in the two plates given by

F = qE

where q is the charge of the particle and E is the electric field created by the charges in the plates at the spot where the particle is currently located. This is a vector relation, which means that the force acting on the particle is entirely vertical - the particle will deflect towards one of the plates. In the horizontal direction, on the other hand, the particle's motion will not be affected. That is, the horizontal component of the particle's velocity will remain

$$v_x = 0.4 \cdot 10^6 \frac{m}{s}$$

It will traverse a horizontal distance of 22 cm in

$$t = \frac{\frac{22}{100}}{0.4 \cdot 10^6} = 5.5E - 07s$$

where we have divided by 100 to convert the distance into m for unit consistency

$$\boxed{t = 5.5E - 07s}$$

Part (b) **When the particle exits the region between the plates, what will be the magnitude of its vertical displacement from its entry height, in millimeters?**

Near the central axis of a flat, charged conductor, the electric field can be well-approximated as pointing normal to the conductor's surface. In the case of two oppositely-charged plates, the strength of the field remains a constant in this region, regardless of how close to either plate we look. We are concerned with the motion of a particle in just such a region.

Since the particle has a charge, it will have a force exerted on it from the charges in the two plates given by

F = qE

where q is the charge of the particle and E is the electric field created by the charges in the plates at the spot where the particle is currently located. This is a vector relation, which means that the force acting on the particle is entirely vertical - the particle will deflect towards one of the plates.

In part a, we found that it will be traveling for

$$t = 5.5E - 07s$$

In this part, we ask how much it will deflect in that amount of time. From Newton's second law, we solve for the acceleration of the particle:

F = ma

qE = ma

$$a = \frac{qE}{m} = \frac{2 \cdot 1.6 \cdot 10^{-19} \cdot 31 \cdot 1000}{5.1 \cdot 10^{-26}} = 1945.1\text{E}8 \quad \frac{m}{s^2}$$

Since a is defined to be the time derivative of velocity and velocity of position, and since the particle starts with no initial vertical velocity, the displacement of the particle is given by

$$y = \frac{1}{2}at^2$$

Substituting in the values found yields

$$y = \frac{1}{2} \cdot 194509803921.569 \cdot (5.5E - 07)^2 =$$

$0.02942m$

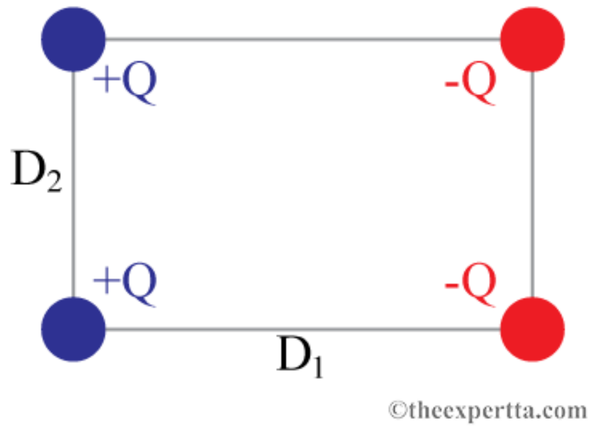
The problem requests the value to be in mm, not m, so we multiply this answer by 1,000

$$y = 29.42mm$$

$$\boxed{y = 29.42mm}$$

Problem 9 - 22.3.8 :

Four point charges of equal magnitude Q = *35* nC are placed on the corners of a rectangle of sides D_1 = *15* cm and D_2 = *6* cm. The charges on the left side of the rectangle are positive while the charges on the right side of the rectangle are negative. Refer to the figure.



Part (a) **Which of the following represents a free-body diagram for the charge on the lower left hand corner of the rectangle?**

We are interested in the forces acting on the positive charge on the lower left corner of the rectangle There are three forces acting on it that are not equal in magnitude. The positive charge on the upper left corner exerts a force in the negative y direction. The negative charge at the lower right exerts a force in the positive x direction, and the negative charge on the upper right exerts a force along the diagonal of the rectangle that points toward the upper right. The free-body diagram is shown here.

Part (b) **Choose an expression for the horizontal component of the net force acting on the charge located at the lower left corner of the rectangle in terms of the charges, the given distances, and the Coulomb constant. Use a coordinate system in which the positive direction is to the right and upwards.**

Of the three forces acting on the charge on the lower left corner, only two have components in the x-direction.

$$\boxed{F_x = F_2 cos\left(\theta\right) + F_3}$$

The cosine of the angle is equal to the side adjacent to the angle divided by the hypotenuse, which is found using the Pythagorean theorem.

$$\begin{aligned} F_x &= F_2 cos\left(\theta\right) + F_3 \\ &= \frac{kQ^2}{\left(\sqrt{D_1^2+D_2^2}\right)^2}\frac{D_1}{\sqrt{D_1^2+D_2^2}} + \frac{kQ^2}{D_1^2} \\ &= kQ^2\left[\frac{1}{\left(D_1^2+D_2^2\right)}\frac{D_1}{\sqrt{D_1^2+D_2^2}} + \frac{1}{D_1^2}\right] \end{aligned}$$

$$\boxed{F_x = kQ^2\left[\frac{1}{D_1^2} + \frac{D_1}{\left(D_1^2+D_2^2\right)^{\frac{3}{2}}}\right]}$$

Part (c) **Calculate the value of the horizontal component of the net force, in newtons, on the charge located at the lower left corner of the rectangle.**

Using the expression derived in part b,

$$\begin{aligned} F_x &= kQ^2\left[\frac{1}{D_1^2} + \frac{D_1}{\left(D_1^2+D_2^2\right)^{\frac{3}{2}}}\right] \\ &= \left(8.99\times10^9\,\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(35\times10^{-9}\text{ C}\right)^2\left[\frac{1}{\left(15\times10^{-2}\text{ m}\right)^2} + \frac{\left(15\times10^{-2}\text{ m}\right)}{\left(\left(15\times10^{-2}\text{ m}\right)^2+\left(6\times10^{-2}\text{ m}\right)^2\right)^{\frac{3}{2}}}\right] \end{aligned}$$

$$\boxed{F_x = 0.0008812\ \text{ N}}$$

Part (d) **Enter an expression for the vertical component of the net force acting on the charge located at the lower left corner of the rectangle in terms of the charges, the given distances, and the Coulomb constant. Assume up is positive.**

Of the three forces acting on the charge on the lower left corner, only two have components in the y-direction.

$$\boxed{F_y = F_2 sin\left(\theta\right) - F_1}$$

The sine of the angle is equal to the side opposite the angle divided by the hypotenuse, which is found using the Pythagorean theorem.

$$\begin{aligned} F_y &= F_2 sin\left(\theta\right) - F_1 \\ &= \frac{kQ^2}{\left(\sqrt{D_1^2+D_2^2}\right)^2}\frac{D_2}{\sqrt{D_1^2+D_2^2}} - \frac{kQ^2}{D_2^2} \end{aligned}$$

$$\boxed{F_y = kQ^2\left[\frac{D_2}{\left(D_1^2+D_2^2\right)^{\frac{3}{2}}} - \frac{1}{D_2^2}\right]}$$

Part (e) **Calculate the value of the vertical component of the net force, in newtons.**

Using the expression derived in part d, we find

$$\begin{aligned} F_y &= kQ^2\left[\frac{D_2}{\left(D_1^2+D_2^2\right)^{\frac{3}{2}}} - \frac{1}{D_2^2}\right] \\ &= \left(8.99\times10^9\,\frac{\text{N}\cdot\text{m}^2}{textC^2}\right)\left(35\times10^{-9}\text{ C}\right)^2\left[\frac{\left(6\times10^{-2}\text{ m}\right)}{\left(\left(15\times10^{-2}\text{ m}\right)^2+\left(6\times10^{-2}\text{ m}\right)^2\right)^{\frac{3}{2}}} - \frac{1}{\left(6\times10^{-2}\text{ m}\right)^2}\right] \end{aligned}$$

$$\boxed{F_y = -0.00290239098961614\ \text{ N}}$$

Part (f) **Calculate the magnitude of the net force, in newtons, on the charge located at the lower left corner of the rectangle.**

Once the components are known, the total force can be found.

$$\begin{aligned} F &= \sqrt{F_x^2 + F_y^2} \\ &= \sqrt{\left(kQ^2\left(\frac{D_1}{\left(D_1^2+D_2^2\right)^{\frac{3}{2}}} + \frac{1}{D_1^2}\right)\right)^2 + kQ^2\left(\frac{D_2}{\left(D_1^2+D_2^2\right)^{\frac{3}{2}}} + \frac{1}{D_2^2}\right)^2} \\ &= kQ^2\sqrt{\left(\left(\frac{D_1}{\left(D_1^2+D_2^2\right)^{\frac{3}{2}}} + \frac{1}{D_1^2}\right)\right)^2 + \left(\frac{D_2}{\left(D_1^2+D_2^2\right)^{\frac{3}{2}}} + \frac{1}{D_2^2}\right)^2} \\ &= \left(8.99\times10^9\,\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(35\times10^{-9}\text{ C}\right)^2\sqrt{\left(\left(\frac{\left(15\times10^{-2}\text{ m}\right)}{\left(\left(15\times10^{-2}\text{ m}\right)^2+\left(6\times10^{-2}\text{ m}\right)^2\right)^{\frac{3}{2}}} + \frac{1}{\left(15\times10^{-2}\text{ m}\right)^2}\right)\right)^2 + \left(\frac{\left(6\times10^{-2}\text{ m}\right)}{\left(\left(15\times10^{-2}\text{ m}\right)^2+\left(6\times10^{-2}\text{ m}\right)^2\right)^{\frac{3}{2}}} + \frac{1}{\left(6\times10^{-2}\text{ m}\right)^2}\right)^2} \end{aligned}$$

$$\boxed{F = 0.003033\ \text{ N}}$$

Part (g) **Calculate the angle, in degrees between -180° and +180°, that the net force makes, measured from the positive horizontal direction.**

Once the components are known, the angle the total force makes with respect to the positive x-axis can be found.

$$\theta = tan^{-1}\left(\frac{F_y}{F_x}\right)$$

$$= tan^{-1}\left(\frac{kQ^2\left[\frac{D_2}{\left(D_1^2+D_2^2\right)^{\frac{3}{2}}}-\frac{1}{D_2^2}\right]}{kQ^2\left[\frac{1}{D_1^2}+\frac{D_1}{\left(D_1^2+D_2^2\right)^{\frac{3}{2}}}\right]}\right)$$

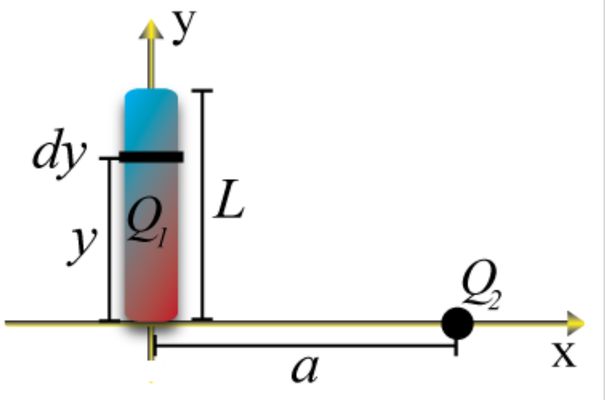
$$= tan^{-1}\left(\frac{\left[\frac{D_2}{\left(D_1^2+D_2^2\right)^{\frac{3}{2}}}-\frac{1}{D_2^2}\right]}{\left[\frac{1}{D_1^2}+\frac{D_1}{\left(D_1^2+D_2^2\right)^{\frac{3}{2}}}\right]}\right)$$

$$= tan^{-1}\left(\frac{\left[\frac{\left(6\times10^{-2}\text{ m}\right)}{\left(\left(15\times10^{-2}\text{ m}\right)^2+\left(6\times10^{-2}\text{ m}\right)^2\right)^{\frac{3}{2}}}-\frac{1}{\left(6\times10^{-2}\text{ m}\right)^2}\right]}{\left[\frac{1}{\left(15\times10^{-2}\text{ m}\right)^2}+\frac{\left(15\times10^{-2}\text{ m}\right)}{\left(\left(15\times10^{-2}\text{ m}\right)^2+\left(6\times10^{-2}\text{ m}\right)^2\right)^{\frac{3}{2}}}\right]}\right)\left(\frac{180^\circ}{\pi\text{ radians}}\right)$$

$$\boxed{\theta = -73.111\ \text{degrees}}$$

Problem 10 - 22.3.11 :

A thin rod of length L = ***1.2*** m lies along the positive y-axis with one end at the origin. The rod carries a uniformly distributed charge of Q_1 = ***2.2*** μC. A point charge Q_2 = ***10.2*** μC is located on the positive x-axis a distance a = ***0.5*** m from the origin. Refer to the figure.



Part (a) **Consider a thin slice of the rod of thickness *dy* located a distance *y* away from the origin. What is the direction of the force on the point charge due to the charge on this thin slice?**

First, let's check the signs of our charges. Q_1 and Q_2 are both positively charged, so they will repel each other. Since any slice of our rod will be partially above the x-axis, it will exert a force both to the right and downward. This means that the force vector will be pointing

Below the positive x-axis

Part (b) **Choose the correct equation for *x*-component of the force, dF_x, on the point charge due to the thin slice of the rod.**

Let's work on assembling our equation one piece at a time. First, let's look at Coulomb's law in vector form.

$$F = \frac{kq_1q_2}{r^2}r_{12}$$

Now, Coulomb's constant is going to be the same in both cases, so we don't need to change that. q_2 is just going to be Q_2 so no major change is needed there, either. Since we are dealing with small slices, and only with force in the x-direction, the F from the normal version of the equation will be a dF_x in this case, which isn't a major change. q_1 and r are a bit trickier, though. Since the rod has charge uniformly distributed, the force exhibited by a small slice of it will not be the entire value of Q_1. Instead, we need to figure out the charge per meter and multiply that by the height of dy in order to figure out how much charge dy has.

$$q_1 = \frac{Q_1}{L}dy$$

With q_1 solved for, we now need to work on finding a value for r, or the total distance between the point charge and the slice of the rod we are examining. Given that the height of our slice is y and the point charge is located on the x-axis a distance a away, we can work this out to

$$r = \sqrt{a^2 + y^2}$$

$$r^2 = a^2 + y^2$$

Now, let's look at the unit vector r_{12}. In our case, we only want the force in the x-direction, so we'll only be using the x-component of r_{12}. This is given by

$$\frac{(x_2 - x_1)}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

Now, we know $x_1 = 0$, $x_2 = a$, $y_1 = y$, and $y_2 = 0$ based on the position of dy and Q_2. Let's plug these values in to the equation above.

$$\frac{(a - 0)}{\sqrt{(a - 0)^2 + (0 - y)^2}}$$

$$\frac{a}{\sqrt{a^2 + y^2}}$$

Now that we have all these components of our equation, let's plug them all in to our original equation of Coulomb's law.

$$dF_x = \frac{k\frac{Q_1}{L}dyQ_2}{a^2+y^2} \cdot \frac{a}{\sqrt{a^2+y^2}}$$

$$dF_x = \frac{kQ_1dyQ_2a}{L\left(a^2+y^2\right)\sqrt{a^2+y^2}}$$

Now we can simplify this a bit by combining our $\left(a^2+y^2\right)$ terms and rearranging the order of some of our variables. This gives us a final result of

$$\boxed{dF_x = \frac{kQ_1Q_2ady}{L(a^2+y^2)^{\left(\frac{3}{2}\right)}}}$$

Part (c) **Integrate the correct expression in part (b) over the length of the rod to find the *x*-component of the net force and calculate its value, in newtons.**

Looking at our equation from part (b), we can see that we are going to need to make some kind of substitution to evaluate it. We have the option of either solving an indefinite integral before substituting back to our original variable and applying our limits of integration at the very end, or working it as a definite integral and changing our limits of integration when we make our substitution. While both methods work equally well, we will be using the first method in this solution.

The first thing that we need to do is to determine what to use for our substitution. In this case, let's use

$$y = a \cdot tan\left(u\right)$$

$$dy = a \cdot sec^2\left(u\right)du$$

Now, substituting these values into our equation, we get

$$F_x = \int \frac{kQ_1Q_2a}{L\left(a^2+\left(atan\left(u\right)\right)^2\right)^{\left(\frac{3}{2}\right)}} \cdot asec^2\left(u\right)du$$

$$F_x = \int \frac{kQ_1Q_2a^2sec^2\left(u\right)du}{L(a^2+a^2tan^2\left(u\right))^{\left(\frac{3}{2}\right)}}$$

$$F_x = \int \frac{kQ_1Q_2a^2sec^2\left(u\right)du}{L(a^2\left(1+tan^2\left(u\right)\right))^{\left(\frac{3}{2}\right)}}$$

To simplify this, we will be using the following trigonometric property

$$sec^2\left(u\right) = 1+tan^2\left(u\right)$$

Now, let's continue working on our equation.

$$F_x = \int \frac{kQ_1Q_2a^2sec^2\left(u\right)du}{L(a^2\left(1+tan^2\left(u\right)\right))^{\left(\frac{3}{2}\right)}}$$

$$F_x = \int \frac{kQ_1Q_2a^2sec^2\left(u\right)du}{L(a^2\left(sec^2\left(u\right)\right))^{\left(\frac{3}{2}\right)}}$$

$$F_x = \int \frac{kQ_1Q_2a^2sec^2\left(u\right)du}{La^3 \cdot sec^3\left(u\right)}$$

$$F_x = \int \frac{kQ_1Q_2du}{Lasec\left(u\right)}$$

$$F_x = \int \frac{kQ_1Q_2cos\left(u\right)du}{La}$$

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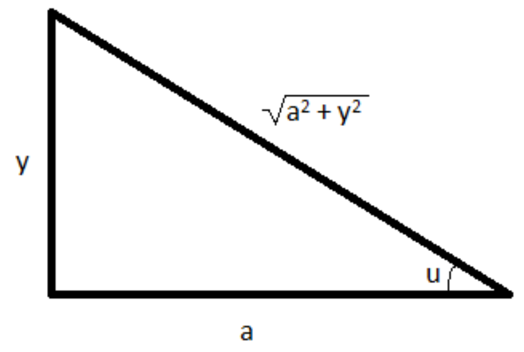
At this point, we need to substitute back to our original variable y and set up our limits of integration. In order to get back to y we first need to set up a triangle. Let's look back at our original substitution and find the triangle we need.

$$a \cdot tan\left(u\right) = y$$

$$tan\left(u\right) = \frac{y}{a}$$

Creating a right triangle for the above equation gives us the following image.



From this image, we can figure out a value for the $sin\,(u)$ from our results. Specifically

$$sin\,(u) = \frac{y}{\sqrt{a^2+y^2}}$$

Plugging this in, we can rewrite our integral as

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Now we just need to find the limits of integration. Since we are going from the bottom of the rod to the top, our limits should be from 0 to L. Solving for these limits, we get

$$F_x = \frac{kQ_1Q_2L}{La\sqrt{a^2+L^2}} - \frac{0}{La\sqrt{a^2}}$$

$$F_x = \frac{kQ_1Q_2}{a\sqrt{a^2+L^2}}$$

Finally, we can plug in our values, remembering to convert our charges from microcoulombs to coulombs

$$F_x = \frac{8.99\cdot 10^9\text{ N }\cdot 2.2\cdot 10^{-6}\text{ C }\cdot 10.2\cdot 10^{-6}\text{ C}}{0.5\text{ m }\cdot\sqrt{(0.5\text{ m })^2+(1.2\text{ m })^2}}$$

$$\boxed{F_x = 0.3104\text{ N}}$$

Part (d) **Choose the correct equation for the y-component of the force, dF_y, on the point charge due to the thin slice of the rod.**

Let's work on assembling our equation one piece at a time. First, let's look at Coulomb's law in vector form.

$$F = \frac{kq_1q_2}{r^2}r_{12}$$

Now, Coulomb's constant is going to be the same in both cases, so we don't need to change that. q_2 is just going to be Q_2 so no major change is needed there, either. Since we are dealing with small slices, and only with force in the y-direction, the F from the normal version of the equation will be a dF_y in this case, which isn't a major change. q_1 and r are a bit trickier, though. Since the rod has charge uniformly distributed, the force exhibited by a small slice of it will not be the entire value of Q_1. Instead, we need to figure out the charge per meter and multiply that by the height of dy in order to figure out how much charge dy has.

$$q_1 = \frac{Q_1}{L}dy$$

With q_1 solved for, we now need to work on finding a value for r, or the total distance between the point charge and the slice of the rod we are examining. Given that the height of our slice is y and the point charge is located on the x-axis a distance a away, we can work this out to

$$r = \sqrt{a^2+y^2}$$

$$r^2 = a^2+y^2$$

Now, let's look at the unit vector r_{12}. In our case, we only want the force in the y-direction, so we'll only be using the y-component of r_{12}. This is given by

$$\frac{(y_2-y_1)}{\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}}$$

Now, we know $x_1 = 0$, $x_2 = a$, $y_1 = y$, and $y_2 = 0$ based on the position of dy and Q_2. Let's plug these values in to the equation above.

$$\frac{(0-y)}{\sqrt{(a-0)^2+(0-y)^2}}$$

$$-\frac{y}{\sqrt{a^2+y^2}}$$

Now that we have all these components of our equation, let's plug them all in to our original equation of Coulomb's law.

$$dF_y = \frac{k\frac{Q_1}{L}dyQ_2}{a^2+y^2} \cdot -\frac{y}{\sqrt{a^2+y^2}}$$

$$dF_y = -\frac{kQ_1dyQ_2y}{L\left(a^2+y^2\right)\sqrt{a^2+y^2}}$$

Now we can simplify this a bit by combining our $\left(a^2+y^2\right)$ terms and rearranging the order of some of our variables. This gives us a final result of

$$\boxed{dF_y = -\frac{kQ_1Q_2ydy}{L(a^2+y^2)^{\left(\frac{3}{2}\right)}}}$$

Part (e) **Integrate the correct expression in part (d) over the length of the rod to find the *y*-component of the net force and calculate its value, in newtons.**

Looking at our equation from part (d), we can see that we are going to need to make some kind of substitution to evaluate it. We have the option of either solving an indefinite integral before substituting back to our original variable and applying our limits of integration at the very end, or working it as a definite integral and changing our limits of integration when we make our substitution. While both methods work equally well, we will be using the first method in this solution.

The first thing that we need to do is to determine what to use for our substitution. In this case, let's use

$$u = a^2 + y^2$$

$$du = 2ydy$$

$$dy = \frac{du}{2y}$$

Now, substituting these values into our equation, we get

$$F_y = -\int \frac{kQ_1Q_2}{2L(u)^{\left(\frac{3}{2}\right)}}du$$

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At this point, we need to substitute back to our original variable.

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Now we just need to find the limits of integration. Since we are going from the bottom of the rod to the top, our limits should be from 0 to L. Solving for these limits, we get

$$F_y = \frac{kQ_1Q_2}{L\sqrt{a^2+L^2}} - \frac{kQ_1Q_2}{L\sqrt{a^2}}$$

$$F_y = \frac{kQ_1Q_2}{L}\left(\frac{1}{\sqrt{a^2+L^2}} - \frac{1}{a}\right)$$

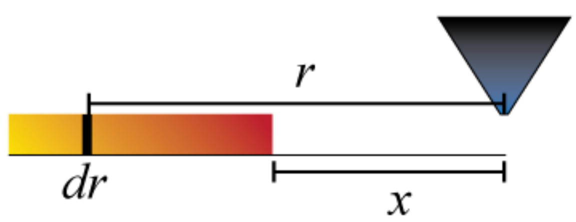
Finally, we can plug in our values, remembering to convert our charges from microcoulombs to coulombs

$$F_y = \frac{8.99 \cdot 10^9 \text{ N } \cdot 2.2 \cdot 10^{-6} \text{ C } \cdot 10.2 \cdot 10^{-6} \text{ C}}{1.2 \text{ m}}\left(\frac{1}{\sqrt{(0.5 \text{ m })^2 + (1.2 \text{ m })^2}} - \frac{1}{0.5 \text{ m}}\right)$$

$$\boxed{F_y = -0.2069 \text{ N}}$$

Problem 11 - 22.3.13 :

A 3-D printer lays down a very long straight line of positively charged plastic with a linear charge density of λ = ***1.1*** μC/m. After the printer has finished a line, the stylus continues moving a distance *x* = ***2.5*** mm in the same direction, as shown in the figure. The minute segment of the plastic line marked in the diagram has a length *dr* and is a distance *r* from the stylus tip. Assume that *r* = 0 is at the stylus tip and the positive *r* direction is towards the charged plastic.



Part (a) **Enter an expression for the charge *dq* on the plastic line segment of length *dr* in terms of defined quantities.**

Based on the definition of linear charge density, the charge on a section of the line is given by the linear charge density multiplied by the length of the segment. The answer to this question is thus

$$\boxed{dq = \lambda dr}$$

Part (b) **Enter an expression for the magnitude of the electrical force, dF_e, exerted by the minute segment of plastic of length *dr* on an electron (charge -*e*) within the stylus at the point *r* = 0. Assume that the line of charged plastic and the electron in question are colinear. Express your answer in terms of defined quantities and physical constants.**

Let's begin with the expression for electrical force.

$$F_e = \frac{kq_1q_2}{r^2}$$

We found an expression for the charge of a small section of the line's charge in part (a), so we can use that again here. The other charge will be the electron's charge of $-e$. The distance from dr to the electron is given by r, so that variable will remain the same. The big thing to watch out for here is the fact that this question asks for magnitude, so we will be dropping the negative sign on the electron's charge out of this equation to ensure that our result is positive. Rewriting the equation above with variables from our system, we get

$$\boxed{dF_e = \frac{ke\lambda dr}{r^2}}$$

Part (c) **Integrate your equation in part (b) to find the magnitude of the force between the electron in the stylus and the entire line of charged plastic. Perform indefinite integration, i.e., don't apply limits yet. Enter the resulting expression with its sign.**

Looking at the equation we found in part (b), we see that the only term that isn't a constant is r. As such, this is a very straightforward integration.

$$dF_e = \frac{ke\lambda dr}{r^2}$$

$$F_e = \int \frac{ke\lambda dr}{r^2}$$

$$\boxed{F_e = -\frac{ke\lambda}{r}}$$

Part (d) **Choose the limits of integration that will result in a correct calculation of the electric force between the electron in the stylus and the entire line of charged plastic.**

Here, we don't actually know the length of the line other than the fact it is very long. As such, we will approximate the line as an infitely long line of charge. Since the line starts at distance x and goes to an infinite distance away, those will be our limits. Further, we want to select the order of our limits of integration so that we get a positive magnitude. This gives us our limits as

$r = x$
to
$r = \infty$

Part (e) **Use your expression in part (c) and your correct choice of limits in part (d) to derive an expression for the magnitude of the electric force between the electron in the stylus and the entire line of charged plastic. Enter the resulting expression.**

Here, we just need to start with our expression from part (c) and evaluate it at the limits we found in part (d).

$$F_e = -\frac{ke\lambda}{r}$$

$$F_e = -ke\lambda\left(\frac{1}{\infty} - \frac{1}{x}\right)$$

$$F_e = -ke\lambda\left(-\frac{1}{x}\right)$$

$$\boxed{F_e = \frac{ke\lambda}{x}}$$

Part (f) **Calculate the magnitude of the electric force, in newtons, between the electron in the stylus and the entire line of charge plastic.**

Here, we just need to plug values into the equation we found in part (e) to find an answer.

$$F_e = \frac{ke\lambda}{x}$$

$$F_e = \frac{8.99 \cdot 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2 \cdot 1.603 \cdot 10^{-19} \text{ C} \cdot 1.1 \cdot 10^{-6} \text{ C/m}}{2.5 \cdot 10^{-3} \text{ m}}$$

$$\boxed{F_e = 6.3408268E - 13 \text{ N}}$$

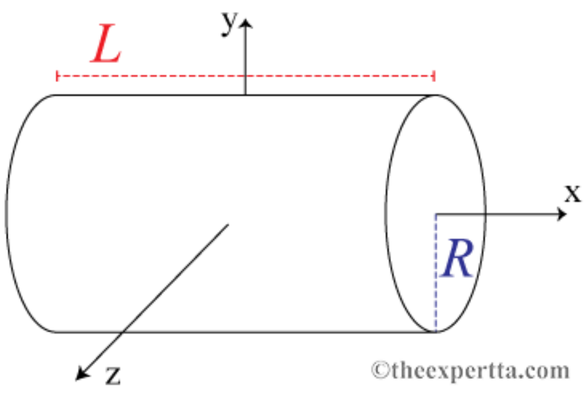
Part (g) **What is the direction of the electric force on the electron in the stylus?**

Given that the electron has a negative charge, and the line of charge is positively charged, they will be attracted to one another. As such, the electric force on the electron will be pointed

Left

Problem 12 - 23.1.8 :

A closed hollow cylinder (i.e., with capped ends) is situated in an electric field given by $\mathbf{E}(u) = E_0(u^5\mathbf{i} + 7\mathbf{j} + 22\mathbf{k})$. The cylinder's axis is on the x-axis with its center at the origin. The cylinder's height is L and its radius is R. Here $u = x/x_0$ is a dimensionless variable, where x_0 sets the scale of the field. Refer to the figure.



Part (a) **Integrate to find an expression for the total electric flux through the cylinder in terms of defined quantities and enter the expression.**

E is constant in the y and z directions so over the curved portion of the cylinder the flux entering and leaving the surface will cancel.

The x component of E changes over the curved surface but is perpendicular to the surface so it generates no flux.

Only the endcaps need to be considered. The y and z components are perpedicular to the endcaps so only the x component needs to be considered.

$$\Phi_{left} = E_{\perp}A = E_0\left(\frac{-\frac{L}{2}}{x_0}\right)^5 \pi R^2 = \frac{E_0\pi R^2 L^5}{32{x_0}^5}$$

Note the result is positive since both E and the normal to the area both point to the left.

$$\Phi_{right} = E_{\perp}A = E_0\left(\frac{\frac{L}{2}}{x_0}\right)^5 \pi R^2 = \frac{E_0\pi R^2 L^5}{32{x_0}^5}$$

$$\Phi = \Phi_{left} + \Phi_{right}$$

$$\boxed{\Phi = \frac{E_0\pi R^2 L^5}{16{x_0}^5}}$$

Part (b) **For L = 4.1 m, R = 0.11 m, E_0 = 2.5 V/m, and x_0 = 1 m, find the value of the electric flux, in units of volt•meter, through the cylinder.**

$$\Phi = \frac{E_0\pi R^2 L^5}{16{x_0}^5} = \left(2.5\frac{V}{m}\right)\pi(0.11m)^2\frac{(4.1m)^5}{16(1m)^5}$$

$$\boxed{\Phi = 6.881Vm}$$

Part (c) **If the electric field is $\mathbf{E}(u) = E_0(323u^2\mathbf{i} + 42\mathbf{j} + 415\mathbf{k})$, enter an expression for the total flux in terms of defined quantities.**

As in part a E is constant in the y and z directions so over the curved portion of the cylinder the flux entering and leaving the surface will cancel.

The x component of E changes over the curved surface but is perpendicular to the surface so it generates no flux.

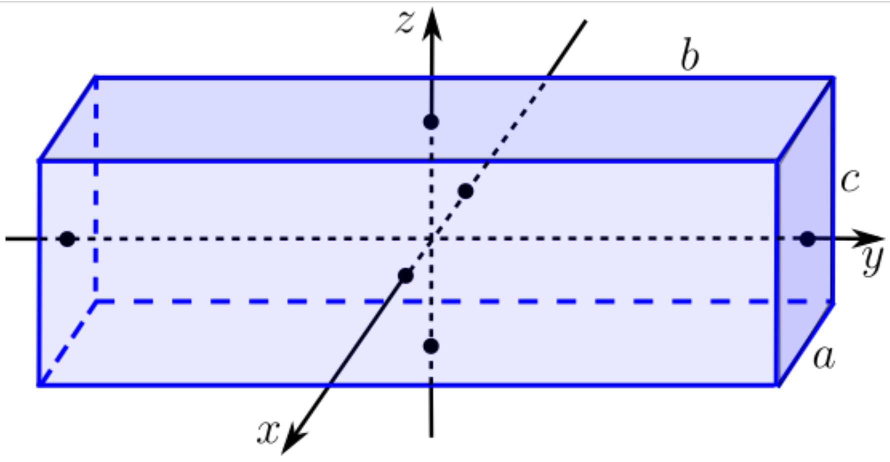
Unlike part a the x component is squared, so the the x component of the field will point to the right for both the left and right endcaps.

With this new field the flux through the endcaps cancel and the total flux is zero.

$$\boxed{\Phi = 0}$$

Problem 13 - 23.2.15 :

A Gaussian surface in the shape of a rectangular box is centered at the origin, and its edges are parallel to the coordinate axes. Edges parallel to the x axis have length a; edges parallel to the y axis have length b; edges parallel to the z axis have length c. A uniform and constant electric field $\mathbf{E} = E_x\hat{\mathbf{i}} + E_y\hat{\mathbf{j}} + E_z\hat{\mathbf{k}}$ fills the region of space which contains the rectangular box.

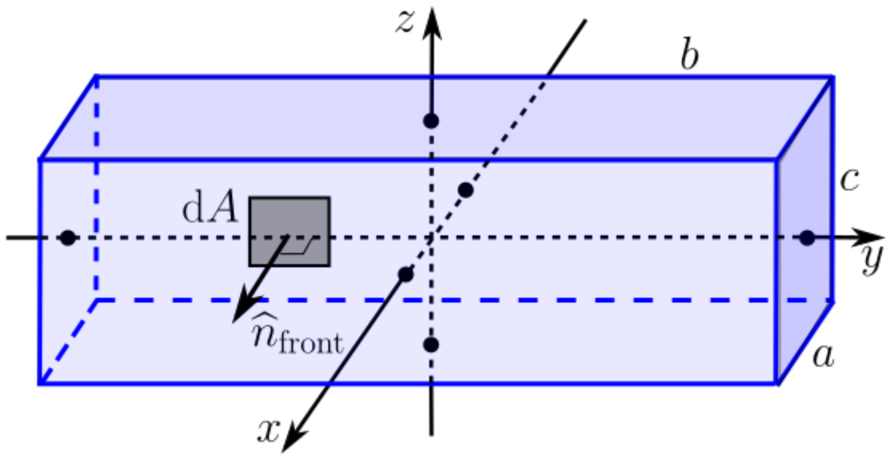


Part (a) **Enter an expression for the outward-pointing normal vector on the front face of the box, $\hat{\mathbf{n}}_{\mathbf{front}}$.**

An area element on the front face of the box

$$\mathrm{d}\vec{A} = \widehat{n}_{\text{front}}\mathrm{d}A$$

is shown on the image below.



The unit normal is parallel to the x axis, hence

$$\boxed{\widehat{n}_{\text{front}} = \hat{i}}$$

Part (b) **Enter an expression for E·dA on the front face of the rectangular box. For the magnitude of dA use dA.**

Calculate the dot product of the area element with electric field.

$$\vec{E}\cdot\mathrm{d}\vec{A} = \left(E_x\ \hat{i} + E_y\ \hat{j} + E_z\ \hat{k}\right)\cdot\left(\mathrm{d}A\ \hat{i}\right)$$

$$\boxed{\vec{E}\cdot\mathrm{d}\vec{A} = E_x\mathrm{d}A}$$

Part (c) **Perform a flux integral over the front face of the box to obtain the outward electric flux, $\Phi_{\mathbf{E,front}}$, through this one face.**

Calculate the flux through the front face.

$$\Phi_{E,\text{front}} = \int_{\text{front}}\vec{E}\cdot\mathrm{d}\vec{A}$$
$$= \int_{\text{front}}E_x\mathrm{d}A$$

Because the field is a constant field, we may factor the constant component E_x out of the integral.

$$\Phi_{E,\text{front}} = E_x\int_{\text{front}}\mathrm{d}A$$
$$= E_x A_{\text{front}}$$

because the meaning of the integral is simply the total area of the front face. Because the front face is a rectangle, its area is the product of its side lengths.

$$\boxed{\Phi_{E,\text{front}} = E_x bc}$$

Part (d) **Enter an expression for the outward-pointing normal vector on the back face of the box, $\hat{\mathbf{n}}_{\mathbf{back}}$.**

The area element dA on the back face of the box is very similar to the area element on the front face except the unit normal is parallel to the $-x$ axis, hence

$$\boxed{\widehat{n}_{\text{back}} = -\hat{i}}$$

Part (e) **Enter an expression for E·dA on the back face of the rectangular box. For the magnitude of dA use dA.**

Calculate the dot product of the area element with electric field.

$$\vec{E}\cdot\mathrm{d}\vec{A} = \left(E_x\ \hat{i} + E_y\ \hat{j} + E_z\ \hat{k}\right)\cdot\left(-\mathrm{d}A\ \hat{i}\right)$$

$$\boxed{\vec{E}\cdot\mathrm{d}\vec{A} = -E_x\mathrm{d}A}$$

Part (f) **Perform a flux integral over the back face of the box to obtain the outward electric flux, $\Phi_{\mathbf{E,back}}$, through this one face.**

Calculate the flux through the back face.

$$\begin{aligned}\Phi_{E,\text{back}} &= \int_{\text{back}} \vec{E}\cdot \mathrm{d}\vec{A} \\ &= \int_{\text{back}} -E_x \mathrm{d}A\end{aligned}$$

Because the field is a constant field, we may factor the constant component $-E_x$ out of the integral.

$$\begin{aligned}\Phi_{E,\text{back}} &= -E_x \int_{\text{back}} \mathrm{d}A \\ &= -E_x A_{\text{back}}\end{aligned}$$

because the meaning of the integral is simply the total area of the back face. Because the back face is a rectangle, its area is the product of its side lengths.

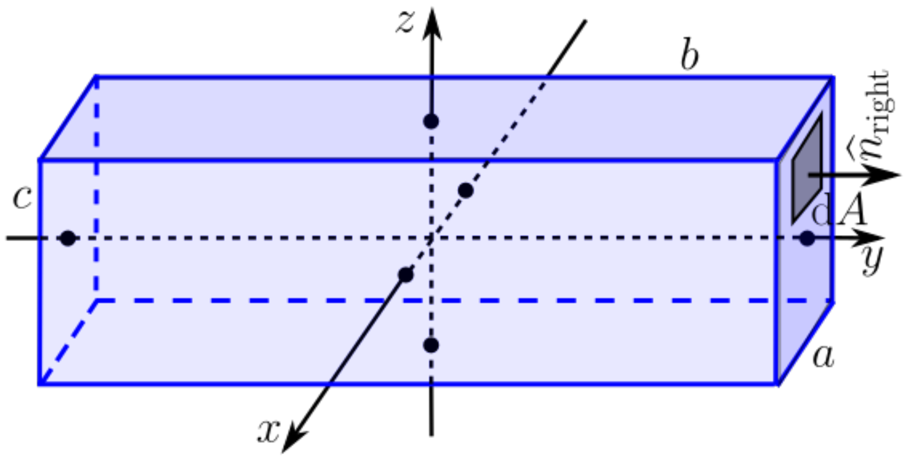
$$\boxed{\Phi_{E,\text{back}} = -E_x bc}$$

Part (g) **Enter an expression for the outward-pointing normal vector on the right face of the box, $\hat{\mathbf{n}}_{\text{right}}$.**

An area element on the right face of the box

$$\mathrm{d}\vec{A} = \widehat{n}_{\text{right}}\mathrm{d}A$$

is shown on the image below.



The unit normal is parallel to the y axis, hence

$$\boxed{\widehat{n}_{\text{right}} = \hat{j}}$$

Part (h) **Enter an expression for E·dA on the right face of the rectangular box. For the magnitude of dA use dA.**

Calculate the dot product of the electric field with the area element.

$$\vec{E}\cdot \mathrm{d}\vec{A} = \left(E_x\,\hat{i} + E_y\,\hat{j} + E_z\,\hat{k}\right)\cdot\left(\mathrm{d}A\,\hat{j}\right)$$

$$\boxed{\vec{E}\cdot \mathrm{d}\vec{A} = E_y \mathrm{d}A}$$

Part (i) **Perform a flux integral over the right face of the box to obtain the outward electric flux, $\Phi_{E,\text{right}}$, through this one face.**

Calculate the flux through the right face.

$$\begin{aligned}\Phi_{E,\text{right}} &= \int_{\text{right}} \vec{E}\cdot \mathrm{d}\vec{A} \\ &= \int_{\text{right}} E_y \mathrm{d}A\end{aligned}$$

Because the field is a constant field, we may factor the constant component E_y out of the integral.

$$\begin{aligned}\Phi_{E,\text{right}} &= E_y \int_{\text{right}} \mathrm{d}A \\ &= E_y A_{\text{right}}\end{aligned}$$

because the meaning of the integral is simply the total area of the right face. Because the right face is a rectangle, its area is the product of its side lengths.

$$\boxed{\Phi_{E,\text{right}} = E_y ac}$$

Part (j) **Enter an expression for the outward-pointing normal vector on the left face of the box, $\hat{\mathbf{n}}_{\text{left}}$.**

The area element dA on the left face of the box is very similar to the area element on the right face except the unit normal is parallel to the $-y$ axis, hence

$$\boxed{\widehat{n}_{\text{left}} = -\hat{j}}$$

Part (k) **Enter an expression for E·dA on the left face of the rectangular box. For the magnitude of dA use dA.**

Calculate the dot product of the electric field with the area element.

$$\vec{E}\cdot \mathrm{d}\vec{A} = \left(E_x\,\hat{i} + E_y\,\hat{j} + E_z\,\hat{k}\right)\cdot\left(-\mathrm{d}A\,\hat{j}\right)$$

$$\boxed{\vec{E}\cdot \mathrm{d}\vec{A} = -E_y \mathrm{d}A}$$

Part (l) **Perform a flux integral over the left face of the box to obtain the outward electric flux, $\Phi_{E,\text{left}}$, through this one face.**

Calculate the flux through the left face.

$$\begin{aligned}\Phi_{E,\text{left}} &= \int_{\text{left}} \vec{E}\cdot \mathrm{d}\vec{A} \\ &= \int_{\text{left}} -E_y \mathrm{d}A\end{aligned}$$

Because the field is a constant field, we may factor the constant component $-E_y$ out of the integral.

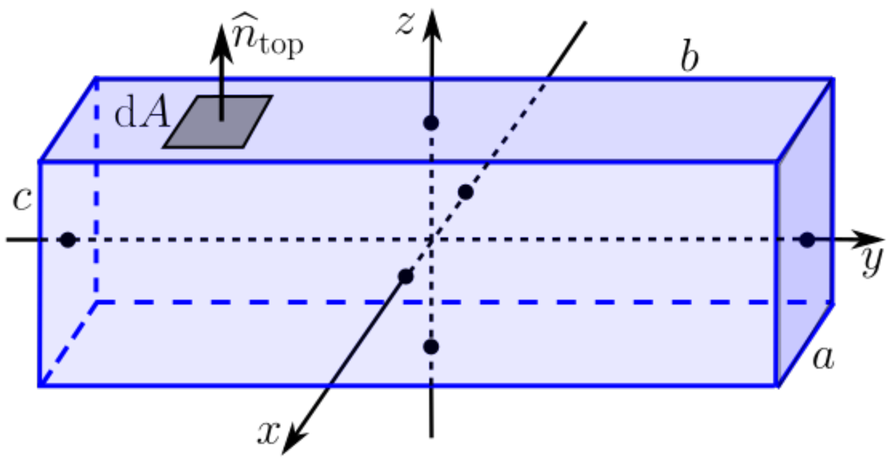
$$\begin{aligned}\Phi_{E,\text{left}} &= -E_y \int_{\text{left}} \mathrm{d}A \\ &= -E_y A_{\text{left}}\end{aligned}$$

because the meaning of the integral is simply the total area of the left face. Because the left face is a rectangle, its area is the product of its side lengths.

$$\boxed{\Phi_{E,\text{left}} = -E_y ac}$$

Part (m) **Perform a flux integral over the top face of the box to obtain the outward electric flux, $\Phi_{E,\text{top}}$, through this one face.**

What was done in three steps for the first four faces is now requested in one step. First we must find the outward-pointing unit normal. An area element on the top face of the box is shown on the image below.



The unit normal is parallel to the z axis, hence

$$\widehat{n}_{\text{top}} = \hat{k}$$

Perform the dot product between the electric field and the area element on the top face.

$$\begin{aligned}\vec{E}\cdot \mathrm{d}\vec{A} &= \left(E_x\ \hat{i} + E_y\ \hat{j} + E_z\ \hat{k}\right)\cdot\left(\mathrm{d}A\ \hat{k}\right)\\ &= E_z \mathrm{d}A\end{aligned}$$

Set up the integral and integrate over the top face.

$$\begin{aligned}\Phi_{E,\text{top}} &= \int_{\text{top}} \vec{E}\cdot \mathrm{d}\vec{A}\\ &= \int_{\text{top}} E_z \mathrm{d}A\end{aligned}$$

Because the field is a constant field, we may factor the constant component E_z out of the integral.

$$\begin{aligned}\Phi_{E,\text{top}} &= E_z \int_{\text{top}} \mathrm{d}A\\ &= E_z A_{\text{top}}\end{aligned}$$

because the meaning of the integral is simply the total area of the top face. Because the top face is a rectangle, its area is the product of its side lengths.

$$\boxed{\Phi_{E,\text{top}} = E_z ab}$$

Part (n) **Perform a flux integral over the bottom face of the box to obtain the outward electric flux, $\Phi_{E,\text{bottom}}$, through this one face.**

What was done in three steps for the first four faces is now requested in one step. First we must find the outward-pointing unit normal. This is similar to the evaluation on the top face, except now unit normal is parallel to the $-z$ axis, hence

$$\widehat{n}_{\text{bottom}} = -\hat{k}$$

Perform the dot product bewteen the electric field and the area element on the bottom face.

$$\begin{aligned}\vec{E}\cdot \mathrm{d}\vec{A} &= \left(E_x\ \hat{i} + E_y\ \hat{j} + E_z\ \hat{k}\right)\cdot\left(-\mathrm{d}A\ \hat{k}\right)\\ &= -E_z \mathrm{d}A\end{aligned}$$

Set up the integral and integrate over the bottom face.

$$\begin{aligned}\Phi_{E,\text{bottom}} &= \int_{\text{bottom}} \vec{E}\cdot \mathrm{d}\vec{A}\\ &= \int_{\text{bottom}} -E_z \mathrm{d}A\end{aligned}$$

Because the field is a constant field, we may factor the constant component $-E_z$ out of the integral.

$$\begin{aligned}\Phi_{E,\text{bottom}} &= -E_z \int_{\text{bottom}} \mathrm{d}A\\ &= -E_z A_{\text{bottom}}\end{aligned}$$

because the meaning of the integral is simply the total area of the bottom face. Because the bottom face is a rectangle, its area is the product of its side lengths.

$$\boxed{\Phi_{E,\text{bottom}} = -E_z ab}$$

Part (o) **Enter an expression, reduced to its most simple form, for the electric flux through the entire closed box, $\Phi_{E,\text{box}}$.**

An integral over any region may be broken into a linear sum of integrals over sub-regions as long as the total region is covered, hence

$$\begin{aligned}\Phi_{E,\text{box}} &= \oint_{\text{box}} \vec{E}\cdot \mathrm{d}\vec{A}\\ &= \int_{\text{front}} \vec{E}\cdot \mathrm{d}\vec{A} + \int_{\text{back}} \vec{E}\cdot \mathrm{d}\vec{A} + \int_{\text{right}} \vec{E}\cdot \mathrm{d}\vec{A} + \int_{\text{left}} \vec{E}\cdot \mathrm{d}\vec{A} + \int_{\text{top}} \vec{E}\cdot \mathrm{d}\vec{A} + \int_{\text{bottom}} \vec{E}\cdot \mathrm{d}\vec{A}\\ &= E_x bc - E_x bc + E_y ac - E_y ac + E_z ab - E_z ab\end{aligned}$$

$$\boxed{\Phi_{E,\text{box}} = 0}$$

Part (p) **Enter an expression, reduced to its most simple form, for the net charged, Q_{enclosed}, contained inside the closed box.**

Gauss' Law relates enclosed charge to the electric flux through a closed surface as

$$\Phi_E = \oint \vec{E}\cdot \mathrm{d}\vec{A} = \frac{Q_{\text{enclosed}}}{\epsilon_0}$$

For the case at hand, we just showed the integral over the closed surface equals zero, hence

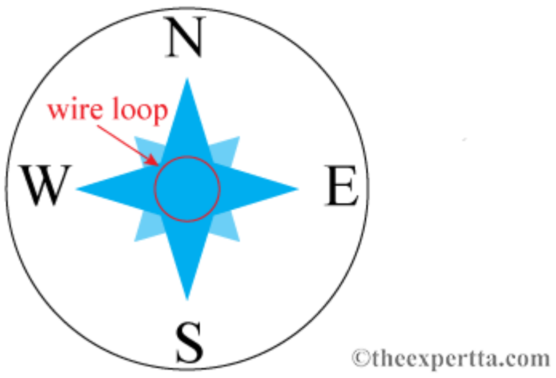
$$0 = \frac{Q_{\text{enclosed}}}{\epsilon_0}$$

and

$Q_{enclosed} = 0$

Problem 14 - 28.3.9 :

You are working on a design for an electromagnetic compass. A single circular loop of wire with an area of A = *1.3* mm^2 is fixed to the face of the compass, and initially lies in a horizontal plane. While active, your circuit is designed to have a counter-clockwise direct current of I = *3.9* μA running through the loop, as seen from above. The compass is in a gimbal which allows the loop to rotate along any axis. You are field testing your compass in an area where the earth's magnetic field has a strength of B = *31* μT.



Randomized Variables

A = *1.3* mm^2
I = *3.9* μA
B = *31* μT
θ =*35*°

Part (a) **Calculate the maximum magnitude of the torque, τ (in N·m), exerted on the current loop by the earth's magnetic field while current is running through the loop.**

The torque exerted by a magnetic field on a current carrying loop is

$$\tau = (\mu\ x\ \mathbf{B})\ \mathrm{N\ m}$$

where τ is the torque vector in N m, μ is the magnetic dipole moment vector in A m^2, and B is the magnetic field vector in T. The magnetic dipole moment vector for a current carrying loop is

$$\mu = I\mathbf{A}$$

where I is the current in A and $\mathbf{A}$ has the area enclosed by the loop as its magnitude and points in a direction perpendicular to the current direction, determined by the right-hand-rule. Substituting in, the magnitude of the torque is

$$\tau = IABsin\,(\theta)$$

where θ is the angle between $\mathbf{A}$ and $\mathbf{B}$ in rad. The maximum occurs when $sin\,(\theta) = 1$, which occurs when the vectors are perpendicular to one another. Therefore, the maximum torque is

$$\tau = IAB$$

Plugging in numbers and converting units as needed,

$$\tau = 3.9 \cdot 10^{-6}\ \mathrm{A} \cdot \left(1.3 \cdot 10^{-3}\ \mathrm{m}\right)^2 \cdot 31 \cdot 10^{-6}\ \mathrm{T}$$

$$\boxed{\tau = 1.5717E - 16\ \mathrm{N\ m}}$$

Part (b) **Towards which direction will the top of the compass (which is pointed out of the page) rotate when the current in the loop is on?**

The torque exerted by a magnetic field on a current carrying loop is

$$\tau = (\mu\ x\ \mathbf{B})\ \mathrm{N\ m}$$

where τ is the torque vector in N m, μ is the magnetic dipole moment vector in A m^2, and B is the magnetic field vector in T. The magnetic dipole moment vector for a current carrying loop is

$$\mu = I\mathbf{A}$$

where I is the current in A and $\mathbf{A}$ has the area enclosed by the loop as its magnitude and points in a direction perpendicular to the current direction, determined by the right-hand-rule. Substituting in, the torque is

$$\tau = I\mathbf{A}\ x\ \mathbf{B}$$

The direction the torque will be such that the direction of A will point in the direction of B. The direction of A is up and the magnetic field of the earth points north.

North

Part (c) **About which of the following axes will the loop rotate when the current is first turned on in the loop?**

In order for the vector A to point north, the loop would have to rotate on an axis perpendicular to north.

East-West

Part (d) **Calculate the magnitude of the torque (in N·m) exerted on the loop when the angle between the normal to the current loop and the earth's magnetic field is θ = *35*°.**

The torque exerted by a magnetic field on a current carrying loop is

$$\tau = (\mu\ x\ \mathbf{B})\ \mathrm{N\ m}$$

where τ is the torque vector in N m, μ is the magnetic dipole moment vector in A m^2, and B is the magnetic field vector in T. The magnetic dipole moment vector for a current carrying loop is

$$\mu = I\mathbf{A}$$

where I is the current in A and $\mathbf{A}$ has the area enclosed by the loop as its magnitude and points in a direction perpendicular to the current direction, determined by the right-hand-rule. Substituting in, the magnitude of the torque is

$$\tau = IABsin\left(\theta\right)$$

where θ is the angle between $\mathbf{A}$ and $\mathbf{B}$ in rad. Plugging in numbers and converting units as needed,

$$\tau = 3.9 \cdot 10^{-6}\ \text{A} \cdot \left(1.3 \cdot 10^{-3}\ \text{m}\right)^2 \cdot 31 \cdot 10^{-6}\ \text{T} \cdot sin\left(35^\circ \cdot \frac{\pi\ \text{rad}}{(180^\circ)}\right)$$

$$\boxed{\tau = 9.0148942071397E - 17\ \text{N m}}$$

Part (e) **A torque of τ = 6.0 × 10^{-14} N·m is required to produce a large enough motion of the compass to be detected with the unaided eye. To produce this effect, you decide to use multiple loops of wire, but keep all other parameters the same. What is the minimum number of loops required?**

The torque exerted by a magnetic field on a current carrying coil is

$$\tau = \left(\mu\ x\ \mathbf{B}\right)\ \text{N m}$$

where τ is the torque vector in N m, μ is the magnetic dipole moment vector in A m^2, and B is the magnetic field vector in T. The magnetic dipole moment vector for a current carrying coil is

$$\mu = NI\mathbf{A}$$

where I is the current in A, N is the number of loops, and $\mathbf{A}$ has the area enclosed by the loop as its magnitude and points in a direction perpendicular to the current direction, determined by the right-hand-rule. Substituting in, the magnitude of the torque is

$$\tau = NIABsin\left(\theta\right)$$

where θ is the angle between $\mathbf{A}$ and $\mathbf{B}$ in rad. The minimum number of loops would occur when the torque is a maximum (the sine term is 1) and the torque equals the minimum value required to create motion detectable with the unaided eye. Therefore,

$$\tau = NIAB$$

$$N = \frac{\tau}{(IAB)}$$

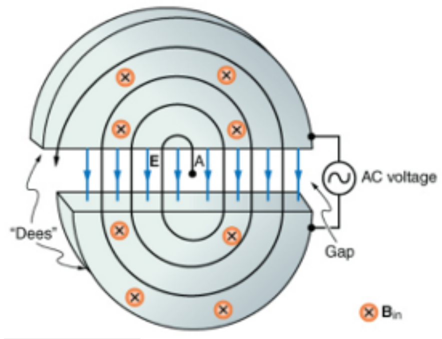
Plugging in numbers and converting units as needed,

$$N_{min} = \frac{\left(6.0 \cdot 10^{-14}\ \text{N m}\right)}{\left(3.9 \cdot 10^{-6}\ \text{A} \cdot \left(1.3 \cdot 10^{-3}\ \text{m}\right)^2 \cdot 31 \cdot 10^{-6}\ \text{T}\right)}$$

$$\boxed{N_{min} = 381.752\ \text{loops}}$$

Problem 15 - 28.5.14 :

A cyclotron accelerates charged particles as shown in the figure. The magnetic field contains the particles within the machine while the electric field in the gap accelerates the particles. Since the particles pass through the electric field in both directions, the frequency of the AC source must match the frequency of the particle motion.



Part (a) **Calculate the frequency, in megahertz, of the accelerating voltage needed for a proton in a *1.05*-T field.**

Our textbook tells us that the cyclotron frequency f is given by

$$f = \frac{eB}{2\pi m}\ ,$$

where e and m denote the proton's charge and mass, respectively, and B denotes the magnetic field strength.

For the proton,

$$e = 1.60 \times 10^{-19}\ \text{C}$$

and

$$m = 1.67 \times 10^{-27}\ \text{kg}\ .$$

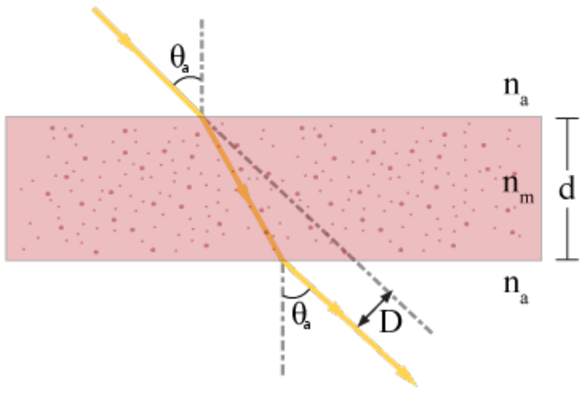
We substitute those values and the given value of the magnetic field strength, taking into account the requested unit for the frequency,

$$f = \frac{\left(1.60 \times 10^{-19}\ \text{C}\right)\left(1.05\ \text{T}\right)}{2\pi\left(1.67 \times 10^{-27}\ \text{kg}\right)}\left(10^{-6}\ \frac{\text{MHz}}{\text{Hz}}\right)\ .$$

$$\boxed{f = 16.011\ \text{MHz}}$$

Problem 16 - 34.4.8 :

A flat slab of material (n_m = *1.8*) is d = *0.15* m thick. A beam of light in air (n_a = 1) is incident on the material with an angle θ_a = *31* degrees with respect to the surface's normal.



Part (a) **Numerically, what is the displacement, *D*, of the beam when it exits the slab?**

To find the value of D, we need to construct a right triangle in the figure. To do that, we define the triangle's vertices. One vertex is at the point where the beam enters the material and we label that point E. Another vertex is at the point where the beam leaves the material and we label that point L. To find the third vertex, we drop a perpendicular from point L to the dashed line that is a continuation of the beam that is incident on the material. The point where the perpendicular intersects the incident beam's continuation line is the third vertex of the triangle and we label that point P.

The right angle is at P. The length of side LP is the requested displacement D.

We still need to know another angle of this triangle and the length of a side. For both we need to find the angle of refraction, with respect to the surface's normal, of the beam entering the material. We denote this angle θ_{m}. We apply Snell's law of refraction and obtain

$$n_{\mathrm{a}} sin\left(\theta_{\mathrm{a}}\right) = n_{\mathrm{m}} sin\left(\theta_{\mathrm{m}}\right),$$

$$\theta_{\mathrm{m}} = sin^{-1}\left(\frac{n_{\mathrm{a}}}{n_{\mathrm{m}}} sin\left(\theta_{\mathrm{a}}\right)\right).$$

Now we can find an expression for the triangle's angle at vertex E, θ_{E}, in terms of given quantities,

$$\theta_{\mathrm{E}} = \theta_{\mathrm{a}} - \theta_{\mathrm{m}} = \theta_{\mathrm{a}} - sin^{-1}\left(\frac{n_{\mathrm{a}}}{n_{\mathrm{m}}} sin\left(\theta_{\mathrm{a}}\right)\right).$$

We can also find the length of side EL of the triangle, which we denote by EL, in terms of given quantities. To do so, consider a second right triangle. Two of its vertices are at points E and L. The third vertex is at the point of intersection of the lower surface of the material with the continuation of the normal to the upper surface at point E. We label that point B. This triangle's right angle is at vertex B. The length of side EB is the depth of the water d. This triangle's angle at vertex E is θ_m. Trigonometry tells us that

$$cos\left(\theta_{\mathrm{m}}\right) = \frac{d}{EL},$$

$$EL = \frac{d}{cos\left(\theta_{\mathrm{m}}\right)} = \frac{d}{cos\left(sin^{-1}\left(\frac{n_{\mathrm{a}}}{n_{\mathrm{m}}} sin\left(\theta_{\mathrm{a}}\right)\right)\right)} = \frac{d}{\sqrt{1-\left(\frac{n_{\mathrm{a}}}{n_{\mathrm{m}}} sin\left(\theta_{\mathrm{a}}\right)\right)^2}}.$$

To find the requested displacement D in terms of given quantities, we note that in our original triangle we have the trigonometric relation

$$sin\left(\theta_{\mathrm{E}}\right) = \frac{D}{EL},$$

$$D = EL sin\left(\theta_{\mathrm{E}}\right) = \frac{d sin\left(\theta_{\mathrm{a}} - sin^{-1}\left(\frac{n_{\mathrm{a}}}{n_{\mathrm{m}}} sin\left(\theta_{\mathrm{a}}\right)\right)\right)}{\sqrt{1-\left(\frac{n_{\mathrm{a}}}{n_{\mathrm{m}}} sin\left(\theta_{\mathrm{a}}\right)\right)^2}}.$$

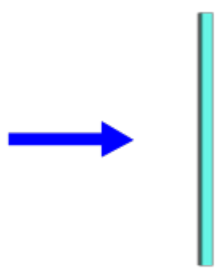
We substitute the given values,

$$D = \frac{(0.15\ \mathrm{m})\, sin\left(31^\circ - sin^{-1}\left(\frac{1}{1.8} sin\left(31^\circ\right)\right)\right)}{\sqrt{1-\left(\frac{1}{1.8} sin\left(31^\circ\right)\right)^2}}.$$

$$\boxed{D = 0.03886\ \mathrm{m}}$$

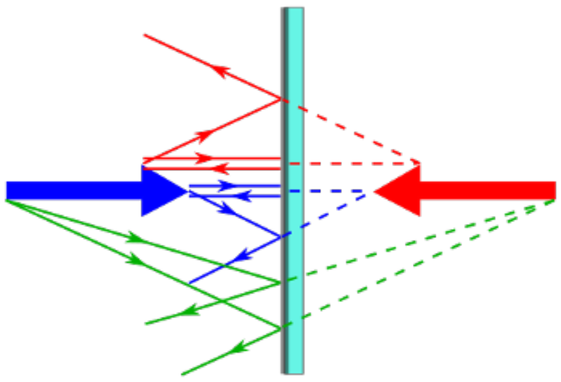
Problem 17 - 35.1.c2 :

An object is placed in front of a simple plane mirror, as shown.



Part (a) **Which diagram best represents the image formed by the plane mirror?**

We know from everyday experience that the image in a plane mirror appears behind the mirror with the same size as the object. Consider your reflection in the bathroom mirror. If you are facing the mirror, you see your face, not the back of your head. We may support this result by examining the diagram below.



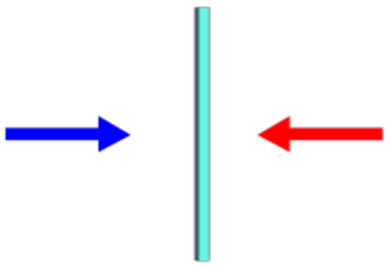
• A ray from the tip of the arrow strikes the mirror at normal incidence and reflects back along the same line. (The rays have been offset slightly to make them both visible, but in reality the rays are along the same line.) The image of the tip will appear symmetrically behind the mirror.

• Similarly, a ray from the the widest point of the arrow strikes the mirror at normal incidence and reflects back along its same line. The image of the this point will appear symmetrically behind the mirror.

• Our first conclusion is that the vertical coordinate of a point on the object and its reflection have the same vertical position.

• To determine the horizontal position, we need two reflected rays which we extrapolate to the image position. We may deduce that a point on the object and its image point are equidistant from the mirror. (Nearest points remain nearest. Farthest points remain the farthest.)

The best choice is:



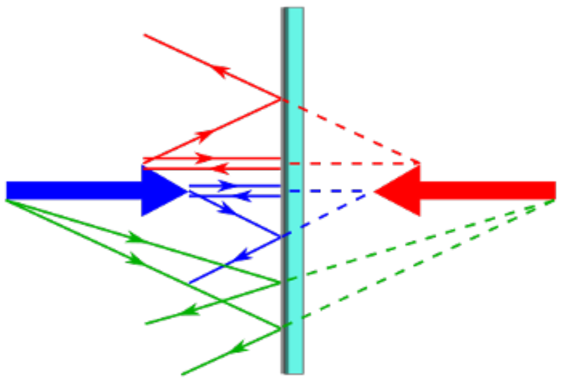
Notes:

• Note that reflection takes place at the front face of the mirror. A thick substrate on the back of the reflective surface does not change the position of the image.

• Infinitely many rays emanate from each point on the object. We tend to focus only on a few rays that have a very simple geometry and easily help construct the solution.

Part (b) **Which statement best describes the image that is formed?**

Refer once again to the ray diagram previously presented.



• A ray from the tip of the arrow strikes the mirror at normal incidence and reflects back along the same line. (The rays have been offset slightly to make them both visible, but in reality the rays are along the same line.) The image of the tip will appear symmetrically behind the mirror.

• Similarly, a ray from the the widest point of the arrow strikes the mirror at normal incidence and reflects back along its same line. The image of the this point will appear symmetrically behind the mirror.

• Our first conclusion is that the vertical coordinate of a point on the object and its reflection have the same vertical position.

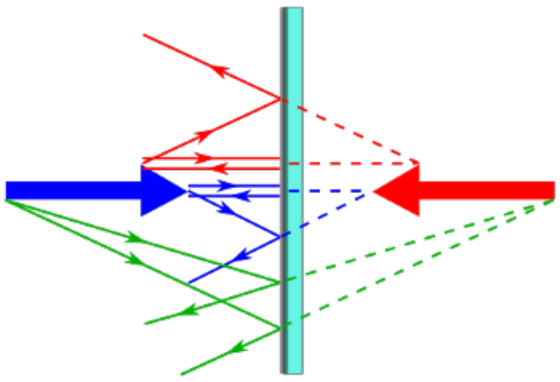
• To determine the horizontal position, we need two reflected rays which we extrapolate to the image position. We may deduce that a point on the object and its image point are equidistant from the mirror. (Nearest points remain nearest. Farthest points remain the farthest.)

The best choice is:

The image is inverted left to right and equal in size to the object.

Part (c) **Which statement best describes the image that is formed?**

Refer once again to the ray diagram previously presented.



• A ray from the tip of the arrow strikes the mirror at normal incidence and reflects back along the same line. (The rays have been offset slightly to make them both visible, but in reality the rays are along the same line.) The image of the tip will appear symmetrically behind the mirror.

• Similarly, a ray from the the widest point of the arrow strikes the mirror at normal incidence and reflects back along its same line. The image of the this point will appear symmetrically behind the mirror.

• Our first conclusion is that the vertical coordinate of a point on the object and its reflection have the same vertical position.

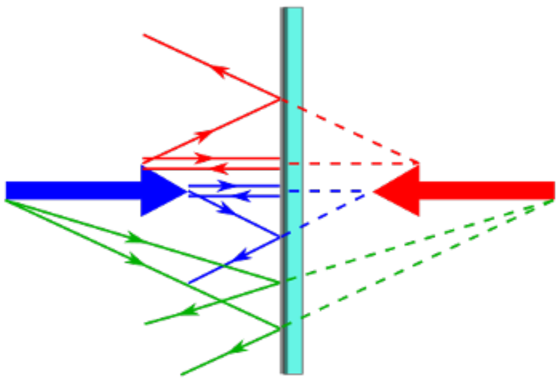
• To determine the horizontal position, we need two reflected rays which we extrapolate to the image position. We may deduce that a point on the object and its image point are equidistant from the mirror. (Nearest points remain nearest. Farthest points remain the farthest.)

The best choice is:

The image and the object are equidistant from the reflecting surface.

Part (d) **Which statement best describes the image that is formed?**

Refer once again to the ray diagram previously presented.



• A ray from the tip of the arrow strikes the mirror at normal incidence and reflects back along the same line. (The rays have been offset slightly to make them both visible, but in reality the rays are along the same line.) The image of the tip will appear symmetrically behind the mirror.

• Similarly, a ray from the the widest point of the arrow strikes the mirror at normal incidence and reflects back along its same line. The image of the this point will appear symmetrically behind the mirror.

• Our first conclusion is that the vertical coordinate of a point on the object and its reflection have the same vertical position.

• To determine the horizontal position, we need two reflected rays which we extrapolate to the image position. We may deduce that a point on the object and its image point are equidistant from the mirror. (Nearest points remain nearest. Farthest points remain the farthest.)

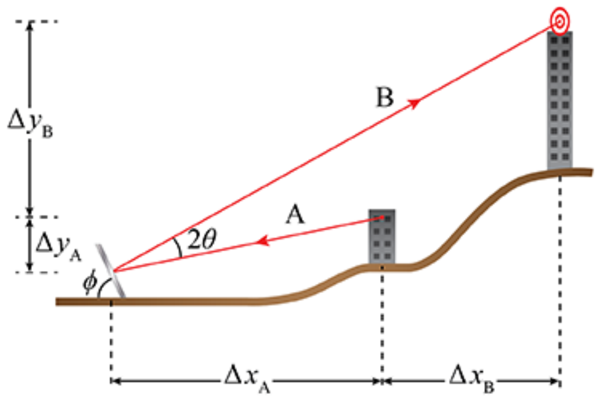
• Note that the reflected rays remain in front of the mirror. The image appears behind the mirror where the reflected rays are extrapolated to a point of convergence.

The best choice is:

The image is virtual because the reflected principal rays may be extrapolated to converge at a common point.

Problem 18 - 35.1.4 :

You mount a laser atop a small building and aim it at a plane mirror located at a horizontal distance Δx_A = *41* m to the left of the laser and a vertical distance Δy_A = *11* m below the laser. Refer to the figure. The laser light incident on the mirror (ray A) is reflected (ray B) and hits a bull's-eye that is at a horizontal distance Δx_B = *6* m behind the laser and a vertical distance Δy_B = *21* m above the laser. The angle of reflection is θ. Let φ denote the acute angle between the mirror and the horizontal, as shown in the figure. In addition, but not shown in the figure, let α_A and denote the angle between ray A and the horizontal and α_B the angle between ray B and the horizontal.



Part (a) **Calculate the angle α_A in degrees.**

A right angle triangle can be drawn tracing the distance the mirror is from the laser along both horizontal and vertical and directly from the mirror to the laser. Therefore,

$$\alpha_A = tan^{-1}\left(\frac{(\Delta y_A)}{(\Delta x_A)}\right)$$

Plugging in numbers and converting units as needed,

$$\alpha_A = tan^{-1}\left(\frac{(11\text{ m})}{(41\text{ m})}\right)$$

$$\boxed{\alpha_A = 15.018\text{ deg}}$$

Part (b) **Calculate the angle α_B in degrees.**

A right angle triangle can be drawn tracing the distance the mirror is from the target along both horizontal and vertical and directly from the mirror to the target. Therefore,

$$\alpha_B = tan^{-1}\left(\frac{(\Delta y_A + \Delta y_B)}{(\Delta x_A + \Delta x_B)}\right)$$

Plugging in numbers and converting units as needed,

$$\alpha_B = tan^{-1}\left(\frac{(11\text{ m} + 21\text{ m})}{(41\text{ m} + 6)}\right)$$

$$\boxed{\alpha_B = 34.249\text{ deg}}$$

Part (c) **Find the angle of reflection, θ, in degrees.**

The 2θ is related to the angles found previously by the expression

$$2\theta = \alpha_B - \alpha_A$$

Therefore

$$\theta = \frac{(\alpha_B - \alpha_A)}{2} = \frac{\left(tan^{-1}\left(\frac{(\Delta y_A+\Delta y_B)}{(\Delta x_A+\Delta x_B)}\right) - tan^{-1}\left(\frac{(\Delta y_A)}{(\Delta x_A)}\right)\right)}{2}$$

Plugging in numbers and converting units as needed,

$$\theta = \frac{\left(tan^{-1}\left(\frac{(11\text{ m}+21\text{ m})}{(41\text{ m}+6)}\right) - tan^{-1}\left(\frac{(11\text{ m})}{(41\text{ m})}\right)\right)}{2}$$

$$\boxed{\theta = 9.615\text{ deg}}$$

Part (d) **Find the acute angle, φ, in degrees, between the mirror and the horizontal.**

By alternate interior angles (created by drawing a horizontal line at the point of reflection),

$$\begin{aligned}\phi =&\ 90 - (\theta + \alpha_A)\\ =&\ 90 - \left(\frac{\left(tan^{-1}\left(\frac{(\Delta y_A+\Delta y_B)}{(\Delta x_A+\Delta x_B)}\right) - tan^{-1}\left(\frac{(\Delta y_A)}{(\Delta x_A)}\right)\right)}{2} + tan^{-1}\left(\frac{(\Delta y_A)}{(\Delta x_A)}\right)\right)\\ =&\ 90 - \left(\frac{\left(tan^{-1}\left(\frac{(\Delta y_A+\Delta y_B)}{(\Delta x_A+\Delta x_B)}\right) + tan^{-1}\left(\frac{(\Delta y_A)}{(\Delta x_A)}\right)\right)}{2}\right)\end{aligned}$$

Plugging in numbers and converting units as needed,

$$\phi = 90^\circ - \frac{\left(tan^{-1}\left(\frac{(11\text{ m}+21\text{ m})}{(41\text{ m}+6)}\right) + tan^{-1}\left(\frac{(11\text{ m})}{(41\text{ m})}\right)\right)}{2}$$

$$\boxed{\phi = 65.366\text{ deg}}$$